Weighing Application Compendium

Density Determination

Simplifying Progress

SARTORIUS

Importance of the Used Formula Symbols

Indices	
a	air
fl	fluid (flowable system)
s	solid
(a)	determined in air
(fl)	determined in a fluid
B	buoyancy
b	bulk
tot	total

Symbol	Size	Unit
m	Mass	kg
V	Volume	m^3
A	Surface	m^2
F	Force	$N = kg \times m/s^2$
$G = m \times g$	Weight force	$N = kg \times m/s^2$
g (gravity)	Acceleration due to	m/s^2
$p = F/A$	Pressure	$Pa = N/m^2$
ρ (rho)	Density	kg/m^3 ; g/cm^3
T	Temperature in Kelvin	K
t	Temperature in °Celsius	°C, t=T-273.15 K
γ (gamma)	Specific weight (old!)	kp/dm^3
α (alpha)	Linear Coefficient of expansion (from solids)	$1/K = K^{-1}$
γ (gamma)	Volume expansion Coefficient (from liquids or gases)	$1/K = K^{-1}$
φ (phi)	Relative humidity	%
π (pi)	Porosity	Volume %

Preliminary Note

Density Measurement

Density as a measurement has a long history that dates back to ancient Greece. The famous Greek scholar Archimedes (287–212 B.C.) was the first person to define a method of measuring density by immersing a solid sample in water and then using the displayed liquid volume and that of a reference sample to determine the sample density. His technique was based on a principle of physics which stated that buoyancy of a sample is directly proportional to the displaced volume, and the higher the density of the sample, the smaller the displaced volume:
$F_{buoyancy} = V_{displaced} \times \rho_{Fluid} \times g$.

The Iranian scholar and polymath Abu Raihan Muhammad al-Biruni (973–1048 A.D.) constructed the first pycnometer, a device used to measure density, and used it to determined the density of various elements. The measuring principle of the pycnometer is based on liquid displacement by the sample to be measured. During the measurement process, the empty vessel (or vessel pre-filled with water) is weighed first. Next, the liquid or sample to be measured is added to the vessel. measured difference in weight is used to calculate the The density of liquids or solids.

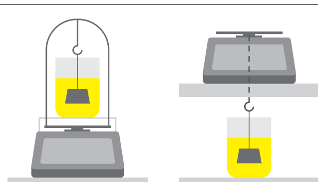
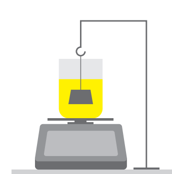
Development of density measurement devices continued into the 18^{th} century. Another popular invention was scaled glass measuring devices such as areometers. These devices operate based on the original Archimedean principle, and are still in use today in select applications, such as alcohol concentration determination.

Today, density measurement is an important part of quality control of raw materials and goods. While the underlying principles remain the same, modern laboratories benefit from advanced instruments with guided protocols that simplify the process for density measurement using the buoyancy or displacement methods or using pycnometers.

Recordkeeping also plays a larger role in today's regulated environments. Thanks to integrated electronic data processing, measured weight values and ambient conditions are digitally logged and the density is calculated automatically. After completing the measurement, the user receives a report that includes all recorded values and the calculated result. These features help to minimizes errors in measurements and recordkeeping, enhancing data reliability in the lab.

Short summary of methods for density determintation using lab balances.

For a detailed comparison of density determination methods see page 35.

	Gravimetric, buoyancy	Gravimetric, displacement	Pycnometer
Methods	 The beaker with liquid stands on a platform or below the scale	 The beaker with liquid is on the scale	 Glass vessel with defined volume
Sample type for:	▪ Solids ▪ Liquids (with glass body)	▪ Pasty substances (with gamma sphere) ▪ Liquids (with glass plummet) Solids	▪ Liquids, dispersions ▪ Powders ▪ Granules
Advantages	▪ Fast procedure ▪ Flexible in terms of sample quantity	▪ Fast procedure	▪ Exact method
Disadvantages	▪ Temperature-sensitive ▪ Sample must be wetted very carefully ▪ Does not tolerate air bubbles	▪ Temperature-sensitive ▪ Requires large sample volume	▪ Temperature-sensitive ▪ High workload ▪ High time expenditure ▪ Does not tolerate air bubbles

Contents

General Principles

Examples for Applications of Density Determination

In many different fields of application, density is used for the identification of material or product properties. Together with other information, density can provide information about possible causes of changes in product properties. Density determination is one of the most frequently used gravimetric methods in the laboratory.

Density can give an indication of changes in the composition of a material or of defects in the form of cracks or cavities in cast parts (so-called blowholes), e.g. in sanitary ceramics or in foundries in the iron and steel industry.

In aluminum foundries, two samples are cast to monitor the melt quality, one under atmospheric pressure, one under a negative pressure of, for example, 80 mbar. After solidification and cooling, the densities of the cast samples are determined; the ratio of the two values provides information on the purity of the melt.

In the case of engineering plastics, the proportion of crystalline phase can be inferred, since crystals have a higher density than the non-crystalline portion due to their greater geometric order. In the case of glass, the density is determined by the cooling rate from the melt in addition to the chemical composition.

In porous materials, the density is influenced by the proportion of pores and determines properties such as the freeze-thaw resistance of roof tiles or the thermal insulation properties of masonry bricks, sand-lime bricks or aerated concrete.

In wine, quality is determined, among other things, by the so-called must weight (in ° Oechsle) - strictly speaking, also a density value, since density is proportional to the concentration of a substance in the solvent (e.g. sugar, salt or alcohol in aqueous solution).

Density is an important parameter in the field of prepackage inspection, when volume data are required on the packages, but the package contents are determined with the aid of balances.

Density Definition

Density ρ is the ratio of mass m and volume V of a quantity of substance. The designations are specified in DIN 1306.

Equation 1

$$\rho = \frac{m}{V}$$

Depending on the field, special compound terms of density are also used:

- Standard density – the density of gases in the standard state (0 °C and 1013 hPa)
- Pouring density – the density of a powder under undefined conditions, e.g. during shipping (according to DIN EN ISO 3252) or the quotient of the mass of uncompacted dry aggregate in a specified measuring vessel divided by the volume of the measuring vessel (according to DIN EN 1097-3)
- Bulk density – the density of a powder in the filled state, determined according to specified test methods, is important for the filling quantity of press molds
- Raw density – the quotient of mass and the volume that includes the voids of a porous material
- Solid density – the quotient of the mass and volume of the solid of a pore-containing body, i.e. excluding the pore volume
- Pure density – still commonly used for solid density
- Relative density – the ratio of a density ρ to a reference density ρ_0 of a reference material, the relative density is a size ratio and has the dimension 1
- Partly, one still finds the indication of the weight or the specific weight, nowadays, it is almost no longer used.

Equation 2

$$\gamma = \frac{G}{V} = \frac{m \times g}{V}$$

In contrast to density, this is an indication of the weight force in relation to the volume, i.e. the density and the specific gravity differ by the factor of the gravitational acceleration g.

Density Units

In the International System of Units, the unit of density is kg/m^3, the most commonly used unit is g/cm^3 - this corresponds to the specification in g/mL. Partly you can also find data in kg/dm^3.

$1\ kg/m^3 = 0.001\ g/cm^3$ or $1\ g/cm^3 = 1\ kg/dm^3 = 1000\ kg/m^3$

Dependence of Density on Temperature

- The density of all solid, liquid and gaseous substances depends on the temperature.
- The density of gaseous substances depends not only on temperature but also on pressure. Gases are compressible at "normal" pressures, which means that the density of air changes with changes in atmospheric pressure.
- The standard density is the density of a gas (or gas mixture) at standard conditions:
  Temperature T = 0 °C, pressure p = 101,325 kPa.
- Generally, the following applies: The higher the temperature, the lower the density. As materials expand with heat, i.e. their volume increases, the density decreases. This effect is less pronounced in solids than in liquids, and is most noticeable in gases.
- The change in density for a given temperature interval can be calculated using the coefficient of thermal expansion; it indicates the change in volume of a substance in relation to temperature. („Temperature Dependence of the Density" see page 37)
- In the following diagrams, the densities of various substances as a function of temperature are calculated and shown graphically - the vertical axes show (except for air) a density range of $0.06\ g/cm^3$ in each case.
- The diagrams show that the temperature dependence of the density of different substances varies. For density measurements this means that – depending on the required measurement accuracy the test temperature must be set very precisely and kept constant.
- For hydrostatic density determination methods, for example, it is more favorable to work with water as a buoyancy medium than with ethanol: When the temperature is increased from 20 °C to 21 °C, for example, the density of the water is reduced by only $0.00021\ g/cm^3$, while that of the ethanol is reduced by $0.00085\ g/cm^3$ – more than four times as much. This means that the temperature constancy must be controlled more carefully than with water, or that a larger error in the density determination must be expected in the case of temperature changes during the experiment.

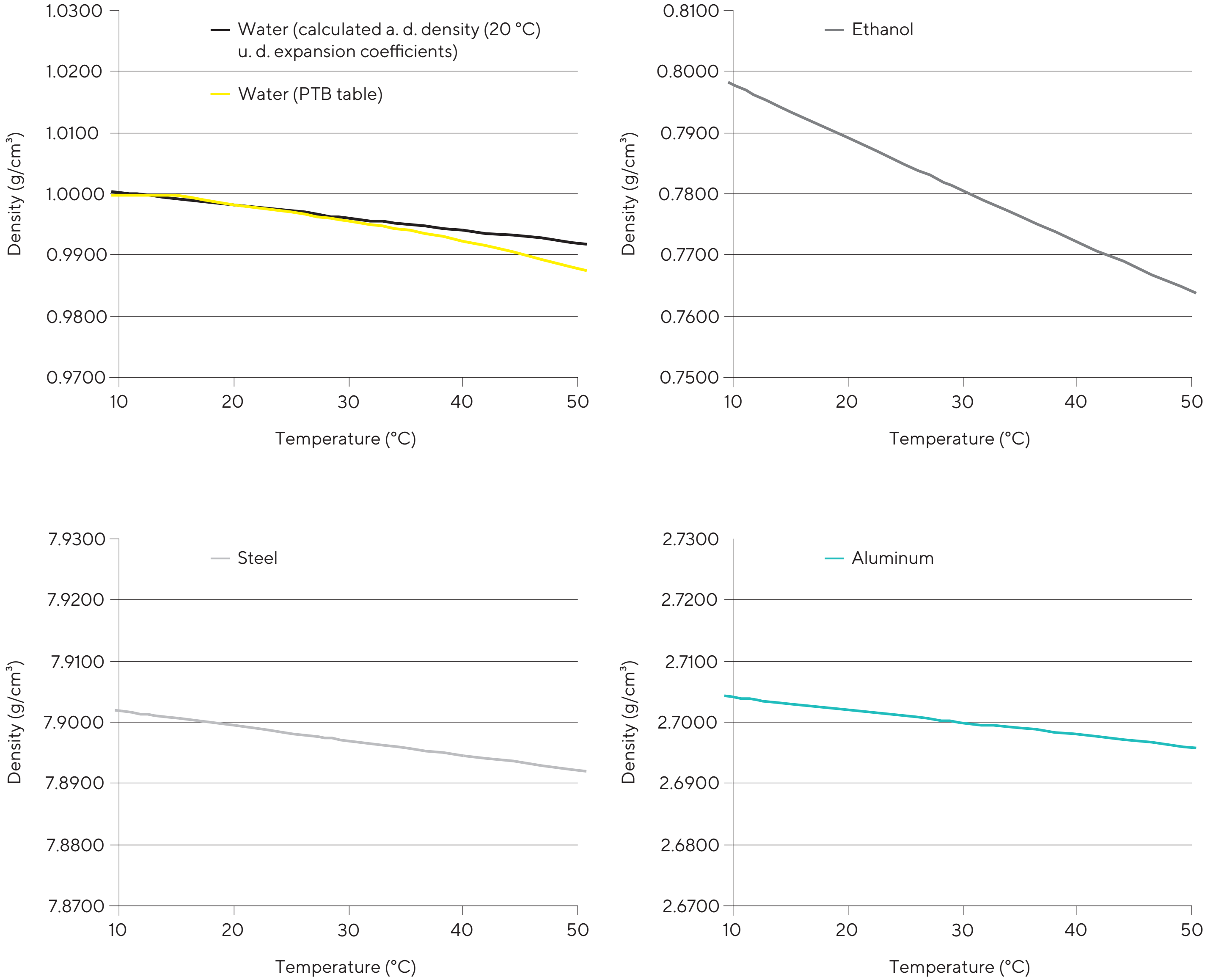
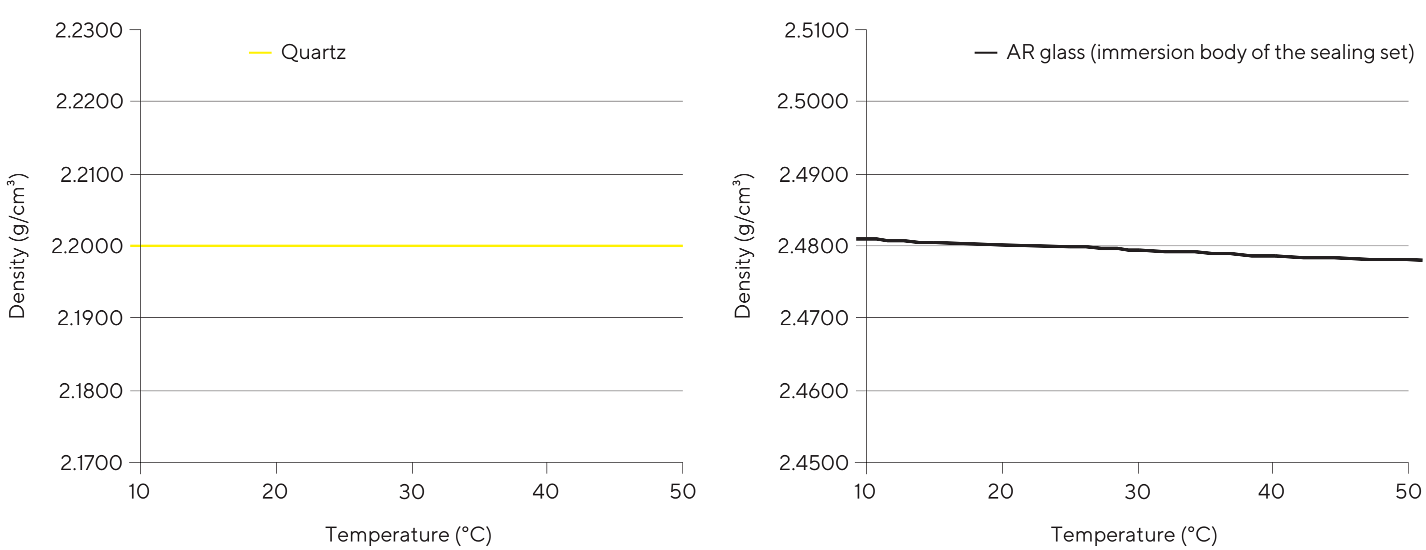
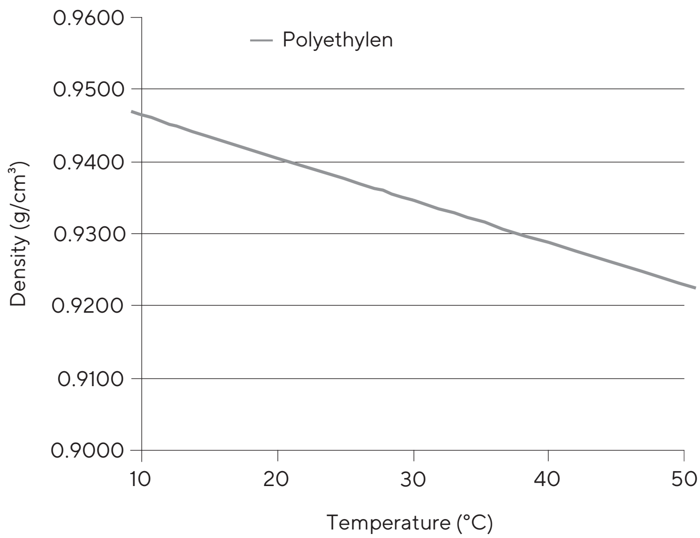


Figure 1: *Temperature Dependence of Density for Water and Ethanol (Above) and for Steel and Aluminum (Below)*





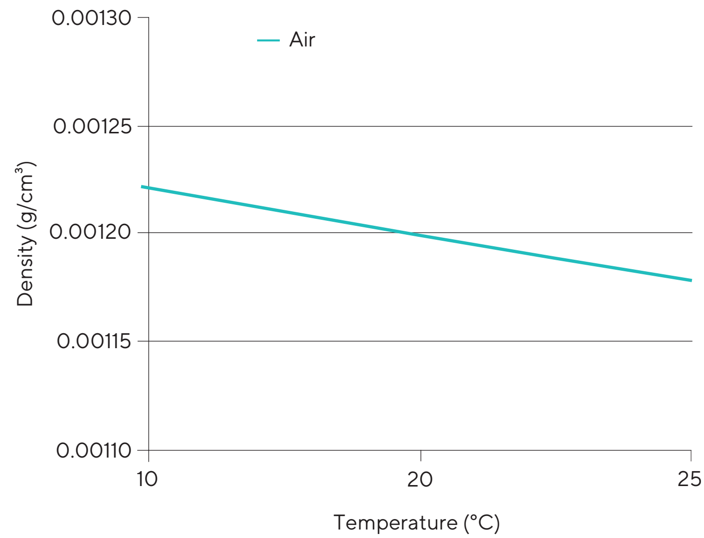


Figure 2: *Temperature Dependence of Density for Glass, Polyethylene and Air*

The Principle of Archimedes

If you want to determine the density of a material, you need the mass and the volume of the sample, according to the definition of density

$$\rho = \frac{m}{V}$$

The mass can be directly determined with the aid of a balance.

In general, the volume cannot be determined directly; exceptions are low accuracy requirements and the presence of simple geometric bodies, such as cubes, cuboids or cylinders, whose volume can be easily calculated from the edge lengths or height and diameter. The volume of liquids can be measured in measuring cylinders or pipettes - the volume of solids can in principle be determined from the rise in the liquid level after immersion of a body in a measuring cylinder filled with water.

Due to the difficulties in the exact determination of the volume, especially of an unevenly shaped body, various methods for determining the density take a "detour". One works on the basis of Archimedes' principle, which describes the relation between forces (or masses), volumes and densities of solids immersed in liquid: From everyday life, everyone knows the effect that bodies appear lighter in water than in air - even your own body in the pool.

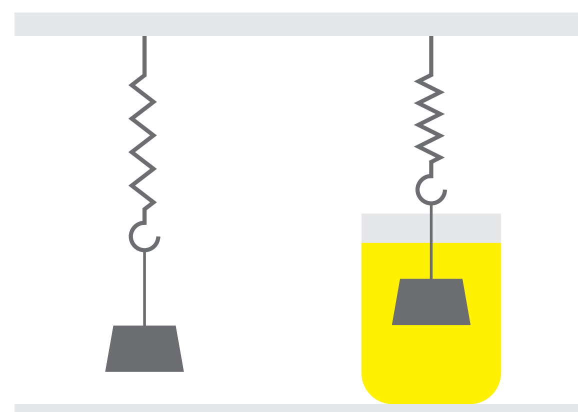


Figure 3: *Force of a Body on a Spring Balance in Air (Left) And in Water (Right)*

The reason for this and the correlations between the individual variables will be explained in the following sections. Forces act on a body immersed in a liquid from all sides as a result of hydrostatic pressure. The forces in the horizontal direction are oppositely equal, i.e. they cancel each other out.

In the vertical direction, the pressure – the deeper one gets below the liquid surface – increases more and more. The pressure caused in a liquid plane by the liquid layer above it is called gravity pressure. It is calculated from the density of the liquid, the height of the liquid column and the acceleration due to gravity:

$$p = \rho_{fl} \times g \times h$$

Accordingly, the force acting on a surface A at depth h is as follows:

Equation 3

$$F = p \times A = \rho_{fl} \times g \times A \times h$$

whereby ρ_{fl} = 1 g/cm³ and g ≅ 10 m/s²

	h	p	A	F
1	1 cm	0.01 N/cm²	1 cm²	0.01 N
2	10 cm	0.1 N/cm²	1 cm²	0.1 N
3	50 cm	0.5 N/cm²	1 cm²	0.5 N

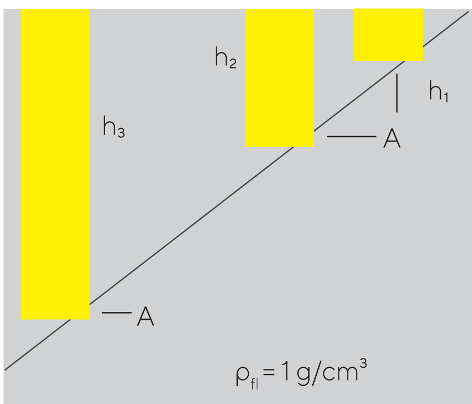


Figure 4: *Course of the Pressure in a Liquid*

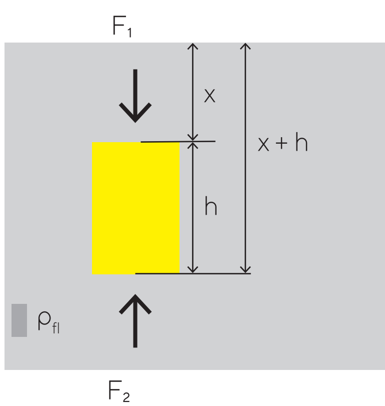


Figure 5: *The Occurrence of the Buoyancy*

The force $F_1 = A \times x \times \rho_{fl} \times g$ acts on the surface of a body of base area A completely immersed in a liquid as a result of the gravitational pressure on the surface.

The force $F_2 = A \times (x + h) \times \rho_{fl} \times g$ acts on the bottom side. The resulting force acting on the body results from the difference of these two forces:

Equation 4

$$F_{res} = F_2 - F_1$$

$$= [A \times (x + h) \times \rho_{fl} \times g] - [A \times x \times \rho_{fl} \times g]$$

$$= A \times (x + h - x) \times \rho_{fl} \times g$$

$$= A \times h \times \rho_{fl} \times g$$

The product of the base area and height of the body is equal to the volume of this body. At the same time, this volume corresponds to the displaced amount of liquid.

Equation 5

$$(A \times h) = V_s - V_{fl}$$

Thus the resulting force becomes

Equation 6

$$F_{res} = V_{fl} \times \rho_{fl} \times g = F_B$$

It is called buoyancy and directly contains the value for the volume you are looking for.

When looking at the force relations on the immersed solid and the liquid element displaced by it, we see the following: In the vertical direction, the downward force G_s acts on the body, as does the upward buoyancy force F_B. The resulting force is the difference of these two forces $F_{res} = G_s - F_B$. The buoyancy force F_B acting on the body corresponds to the weight force G_{fl} of the quantity of liquid displaced by the body.

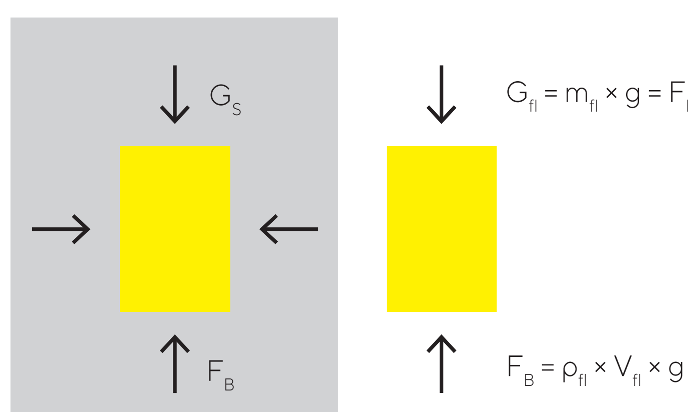


Figure 6: *Regarding Archimedes' Principle – on the Left Solid Immersed in Liquid, on the Right the Liquid Element*

This is

Equation 7

$$G_{fl} = m_{fl} \times g = \rho_{fl} \times V_{fl} \times g$$

When the solid and liquid elements are in equilibrium, the buoyancy force F_B must be equal (in magnitude) to the weight force G_{fl}, i.e.

Equation 8

$$F_B = G_{fl}$$

Thus, buoyancy occurs due to the pressure conditions in a fluid. The buoyancy force is opposite to the weight force of a body immersed in a liquid. This explains that bodies appear lighter in a liquid than in air. Depending on the ratio of the body's weight force to the buoyancy force, the immersed body in the liquid can sink to the bottom, or float.

When the buoyancy force is less than the weight force ($F_B < G_s$), the body sinks to the ground. In this case, the density of the solid is greater than that of the liquid ($\rho_s > \rho_{fl}$).

The widely used method of density determination by the buoyancy method is usually carried out under these conditions.

When the buoyancy force is equal to the weight force ($F_B = G_s$), the body remains completely submerged and floats in the liquid. Since the volume of the body is equal to the volume of the displaced liquid, and the mass of the body is equal to the mass of the displaced liquid, it follows that the densities of solid and liquid are equal. There are various density determination methods that exploit this condition (see page 17).

When the buoyancy force is greater than the weight force ($F_B > G_s$), the body floats, i.e., it rises to the surface of the liquid and becomes only partially submerged. It is immersed in the liquid until the weight of the volume of liquid it displaces is equal to its own weight: In the case of floating, the volume of displaced liquid is smaller than the solid volume ($V_{fl} < V_s$), the density of liquid is greater than the density of solid $\rho_{fl} > \rho_s$. This condition is used for density determination with hydrometers (see page 18).

Gravimetric Methods of Density Determination

Density Determination Methods Based on Archimedes' Principle

The relations described by Archimedes between Mass, volume and density of solids immersed in liquids are used to determine the unknown densities of substances.
The problem with density determination lies in the exact determination of the volume of the sample material. When a solid is completely immersed in a liquid, it is given by the experimental arrangement that the volume of the solid is equal to the volume of the liquid displaced. Under this assumption, the following general relation between the densities and masses of liquids and solids can be derived, in which the volume is no longer explicitly included (for derivation see „Appendix“ page 37):

Equation 9

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{fl}}$$

The unknown density of a solid substance can therefore be determined with an auxiliary liquid of known density and two mass measurement values

Equation 10

$$\rho_{fl} = \rho_s \times \frac{m_{fl}}{m_s} \quad \text{resp.} \quad \rho_{fl} = \frac{m_{fl}}{V_s}$$

Conversely, the density of liquids can be determined from a mass value and the known volume value of the solid using an auxiliary solid.

It is therefore possible to replace volume measurement with mass determinations that can be carried out simply and accurately.

For density measurement, the so-called hydrostatic balance or Mohr's balance is still used in some cases. The Mohr balance, a lever balance, has now been largely replaced by density sets as accessories to laboratory balances. A basic distinction must be made between two hydrostatic weighing methods. Depending on the test arrangement, the measured values displayed by the balance have a different meaning: In the buoyancy method (see Figure 7 and Figure 8), the weight of the solid in the liquid reduced by the buoyancy is determined; in the displacement method (see Figure 9), the weight or mass of the displaced quantity of liquid is determined directly.

Further methods based on Archimedes' principle are density determination with hydrometers (see page 18) and the various levitation methods (see page 17).

Buoyancy Method

The buoyancy method is often used to determine the density of solids and liquids. The apparent weight of a body in a liquid is measured, i.e. the weight minus the buoyancy force, and the density is calculated from this and from the weight in air.

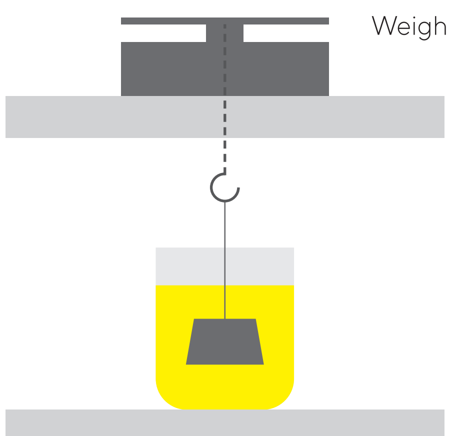


Figure 7: *Principle Test Arrangements for the Buoyancy Method with Underfloor Weighing Equipment*

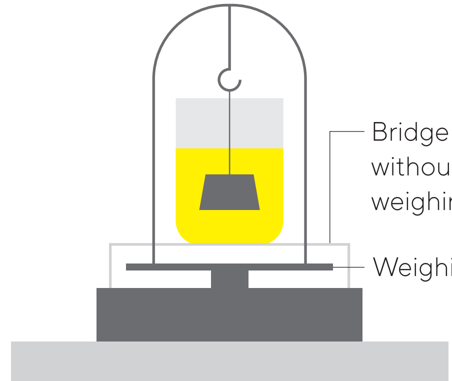


Figure 8: *Principle Test Arrangements for the Buoyancy Method Using a Frame with a Suspension Device for the Immersion Body and Bridge for the Liquid Tank*

In the experimental setup shown in Figure 7 and Figure 8, the reading of the balance corresponds to the mass of the immersed solid reduced by the buoyancy (see Figure 3). That is, considering the equation $\rho_s = \rho_{fl} \times (m_s/m_{fl})$, the mass of the solid weighed in air $m_s = m_{(a)}$ is known. The mass of the liquid m_{fl} is not known directly, but it results from the difference between the weight of the solid in air ($m_{(a)}$) and the weight in liquid ($m_{(fl)}$):

$$m_{fl} = m_{(a)} - m_{(fl)}$$

Thus, equation 2 for the determination of the Solid density becomes:

Equation 11

$$\rho_s = \rho_{fl} \times \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$$

In the case where the liquid density is to be determined, m_{fl} is again to be calculated from the measured values of the solid mass in air and immersed in liquid $m_{fl} = m_{(a)} - m_{(fl)}$ and then inserted into equation 3. The relation for the determination of the liquid density with the buoyancy method is obtained

Equation 12

$$\rho_{fl} = \rho_s \times \frac{m_{(a)}}{m_{(a)} - m_{fl}} = \frac{m_{(a)} - m_{fl}}{V_s}$$

V_s is the known volume of the immersed body, with the help of which the liquid density can be calculated. Thus, the density of a substance can be determined with the help of two weightings.

Displacement Method

With the aid of the displacement method, likewise utilizing Archimedes' principle, the density of solids and liquids can be determined.

In the displacement experiment, the mass of the liquid displaced by the solid is determined directly quantity of liquid displaced by the solid.

A container of liquid stands directly on the weighing pan while the solid is immersed. In this case, the solid is usually suspended from a tripod. When immersed in the liquid, the solid displaces the liquid volume V_{fl} with density ρ_{fl} and mass m_{fl}. The buoyant force acting on the solid is $F_B = \rho_{fl} \times V_{fl} \times g = m_{fl} \times g$. Since the weight of the solid $G_s = m_s \times g$ is held by the stand and does not load the balance, the display of the balance corresponds directly to the liquid mass m_{fl} – provided that the balance was previously tarred with the liquid container.

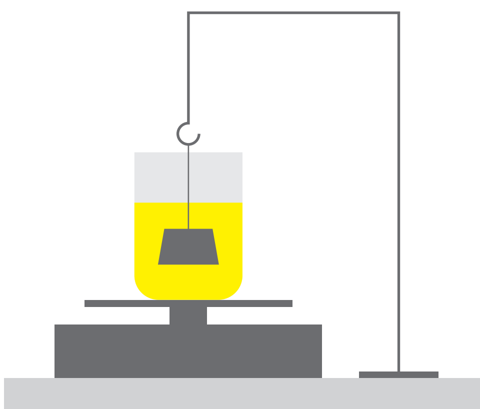


Figure 9: *Principle Test Arrangement for the Displacement Method*

That is, in the case of the displacement method, equations 2 and 3 (see page 11) can be directly applied to the density determination. The following applies to the solid density:

Equation 13

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{fl}}$$

and for the determination of the liquid density

Equation 14

$$\rho_{fl} = \rho_s \times \frac{m_{fl}}{m_s} = \frac{m_{fl}}{V_s}$$

If one uses a plunger with known volume V_s, one can calculate the unknown density ρ_{fl} of a liquid from only one measured value.

Determination of the Density of Air

To convert the weight value into the true mass, one needs the value of the air density.
This can vary in the course of a day by an average of ± 0.05 mg/cm³ relative to the standard density of 1.2 mg/cm³. Therefore, the air density for mass determinations must be determined currently with a relative measurement uncertainty $< 5 \times 10^{-4}$. The air density ρ_a depends on the temperature T, the pressure p and the relative humidity of the air φ. There are approximation equations of different accuracy, with the help of which the air density can be calculated as a function of air pressure, temperature and humidity, possibly also taking into account the CO_2 content of the air.

In addition, with high-resolution balances it is possible to determine the air density (with an error of ≈ 1%). Two different calibrated weights made of materials with different densities (e.g. aluminum and steel) are used. The basis of the determination method is also here the principle of Archimedes. Since air is also made of matter, bodies in air experience a buoyancy force just as they do in a liquid. The same laws apply as described in chapter „The Principle of Archimedes".

If we consider - initially in a vacuum - an aluminum cylinder with the density $\rho_{Al} \approx 2.7$ g/cm³ (Figure 10, left), it is in equilibrium with a weight standard (ρ_N = 8. 000 g/cm³) of the same mass.

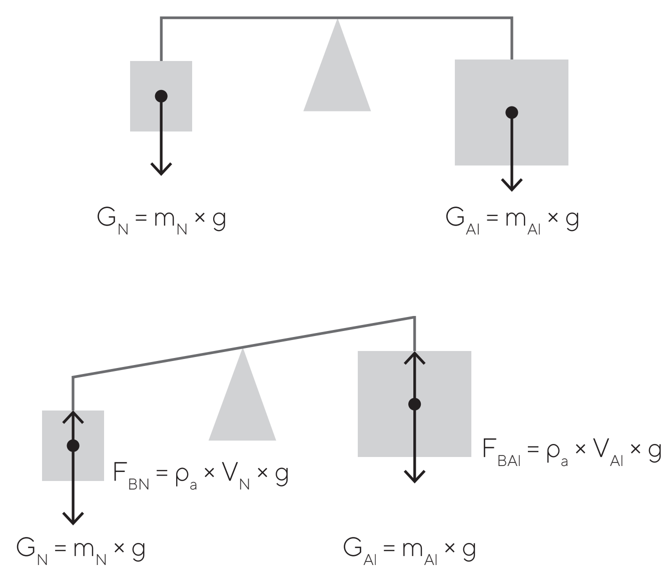


Figure 10: *Influence of Air Buoyancy During Weighing in Vacuum (Above) and in Air (Below)*

Looking at the same arrangement in air (Figure 10, right), the aluminum cylinder and the normal no longer in balance. The cause are the different buoyancy forces - due to the different material densities and therefore different volumes.

To determine which mass mN gives equilibrium to the aluminum cylinder (m_{AL}) in air of density ρ_a, all acting forces are considered in equilibrium:

Equation 15

$$G_N - F_{BN} = G_{Al} - F_{BAl}$$

$$\underbrace{m_N \times g}_{\text{Weight force}} - \underbrace{\rho_a \times V_N \times g}_{\text{Buoyancy force}} = \underbrace{m_{Al} \times g}_{\text{Weight force}} - \underbrace{\rho_a \times V_{Al} \times g}_{\text{Buoyancy force}}$$

After transformation and insertion of

$$V_N = \frac{m_N}{\rho_N} \quad \text{and} \quad V_{Al} = \frac{m_{Al}}{\rho_{Al}}$$

one finally obtains

Equation 16

$$m_{Al} = m_n \times \frac{1 - \frac{\rho_a}{\rho_N}}{1 - \frac{\rho_a}{\rho_{Al}}}$$

m_N is referred to as the weight W. The weight value generally corresponds to the display of the balance.

The weight value

$$W_{Al} = m_{Al} \times \frac{1 - \frac{\rho_a}{\rho_{Al}}}{1 - \frac{\rho_a}{\rho_N}}$$

is not constant, but dependent on the air density – on the weather, so to speak.

This relation applies accordingly to the steel cylinder of the air density determination sets:

$$W_{St} = m_{St} \times \frac{1 - \frac{\rho_a}{\rho_{St}}}{1 - \frac{\rho_a}{\rho_N}}$$

From these two equations a relation can be derived for determining the air density (see Appendix page 38):

Equation 17

$$\rho_a = \frac{m_{Al} \times W_{St} - m_{St} \times W_{Al}}{\frac{m_{Al} \times W_{St}}{\rho_{Al}} - \frac{m_{St} \times W_{Al}}{\rho_{St}}}$$

W_{St} and W_{Al} are the weight values that are currently measured.

m_{St} and m_{Al} are calculated from the conventional weights and the densities of the certified weight pieces calculated according to the relation

Equation 18

$$m_{St} = M_{St} \times \frac{1 - \frac{1.2\ \text{kg/m}^3}{8000\ \text{kg/m}^3}}{1 - \frac{1.2\ \text{kg/m}^3}{\rho_{St}}}$$

and

$$m_{Al} = M_{Al} \times \frac{1 - \frac{1.2\ \text{kg/m}^3}{8000\ \text{kg/m}^3}}{1 - \frac{1.2\ \text{kg/m}^3}{\rho_{Al}}}$$

The conventional weight M of a piece is not the mass of this piece itself, but it corresponds to the mass of the reference weight (mass standard) which is in equilibrium with the considered piece under defined conditions*.

With the conventional weight values of the weights specified in the air density determination set (referred to as characteristic values of the weights), the material densities of the weights and from the current weight values, the air density ρ_a can thus be calculated.

The formula for calculating the air density including the values ρ_{ST} = 8.000 g/cm³ and ρ_{Al} = 2.700 g/cm³ is integrated in the software of some Sartorius balances. The current value of the air density can be stored and is then used to convert the weight values into the actual masses of the material to be weighed – according to the formula derived at the beginning of the chapter

$$m = W \times \frac{1 - \frac{\rho_a}{\rho_N}}{1 - \frac{\rho_a}{\rho_x}}$$

*Temperature T = 20 °C
Density of the mass standard at 20 °C: ρ_N = 8000 kg/m³
Air density ρ_a = 1.2 kg/m³

Density Determination with Pycnometers

Pycnometers are glass or metal vessels with a precisely defined volume for determining the density of liquids and dispersions by simply weighing out the defined volume, but in particular also for determining the density of powders and granules. They are also used to determine the solid density of porous solids, whereby the material must be crushed to such an extent that all pores are opened before the density is determined.

In various fields, differently shaped and standardized pycnometers have become established. During measurements, care must be taken to ensure that all weightings are performed at constant temperature and that no air is trapped either in the liquid or between the particles.

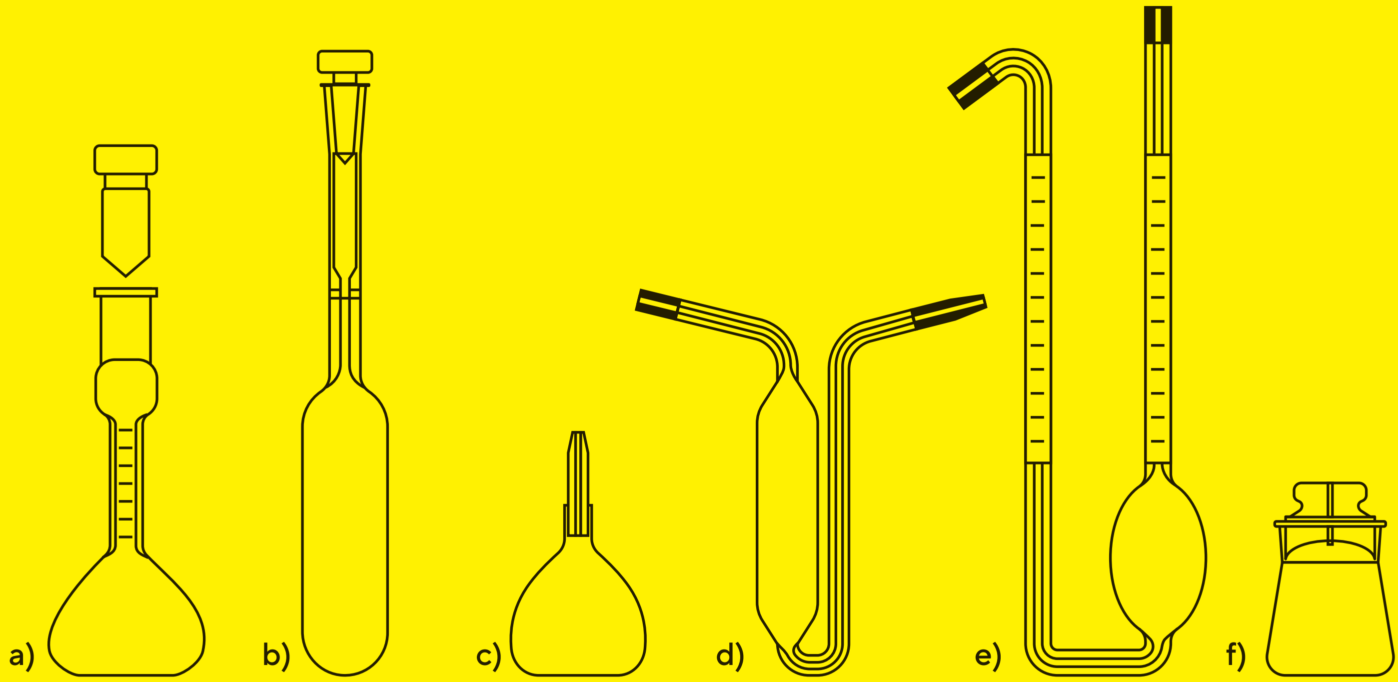


Figure 11: *Examples of Common Glass Pycnometers for Density Determinations on a Laboratory Balance*

Note: pycnometers to Gay-Lussac, DIN EN 12797 (c) and to Hubbard, DIN EN 12806, (f) are used for density determinations of solids; the volume indicated applies to complete filling after insertion of the stopper; the pycnometers to Bingham, DIN 12807, (b) and to Sprengel, ISO 12800, (d) have a ring mark which indicates the volume indicated; the pycnometers to Reischauer, DIN ISO 3507, (a) and Lipkin, DIN ISO 3507, (e) have a balance for reading off the volume.

Weighing a Defined Volume ("Liter Weight")

A particularly simple gravimetric method for determining the density of flowable substances (liquids, powders, disperse systems) is the weighing of a sample quantity with a defined volume. The sample to be determined is filled into a vessel with a known volume and the mass of the sample (after taring) is determined by a single weighing. According to $\rho = m/V$, the density can be easily calculated.

In various industrial sectors, there are different standardized or normed containers, for example conical 1 L vessels for determining the density of casting compounds (slips) in the ceramics industry. In the lime industry, the bulk density of quicklime granules is determined by a standardized method. Both the vessel for holding the sample material and the filling procedure are precisely defined. According to U.S. and British standards, there are cylindrical stainless steel containers, the so-called specific gravity cups, with different volumes and different tolerances of the volume.

Pycnometer Method

The pycnometer method is a very accurate method for determining the density of powders, granules and difficult to flow dispersions. Compared to the buoyancy and displacement method, it is more labor-intensive and significantly more time-consuming.

Here again, the problem of the exact volume determination of a powder sample V_s for the density determination of the solid ρ_s arises. The explicit volume determination of powders or granules is basically circumvented by three weighing operations and the use of an auxiliary liquid of known density.

Equation 19

$$\rho_s = \frac{m_s}{V_s}$$

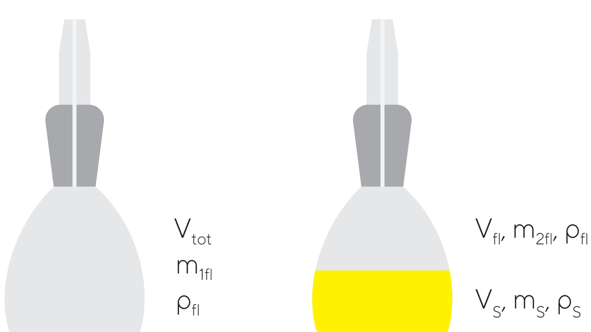


Figure 12: *Pycnometer With Content*

The volume of the solid Vs can only be determined indirectly:

Equation 20

$$V_s = V_{tot} - V_{fl}$$

One proceeds step by step:

- First, the mass of the liquid of the pycnometer completely filled with liquid is determined. Based on the liquid density, the volume of the pycnometer V_{tot} is thus known.

Equation 21

$$V_{tot} = \frac{m_{1fl}}{\rho_{fl}}$$

- Subsequently (after emptying, cleaning, drying and tempering of the pycnometer) the pycnometer filled with sample material to approx. ⅔ is weighed, thus obtaining the mass of the powder used m_s.
- Next, the pycnometer with the sample material is completely filled with the liquid and weighed again. One determines the mass of sample and liquid together $m_{(fl+s)}$.

From this, the mass of the liquid m_{2fl} can be calculated

Equation 22

$$m_{2fl} = m_{(fl+s)} - m_s$$

and thus also the volume V_{fl} of the liquid in the chamber filled with sample material and liquid-filled pycnometer

Equation 23

$$V_{fl} = \frac{m_{2fl}}{\rho_{fl}} = \frac{m_{(fl+s)} - m_s}{\rho_{fl}}$$

The actually interesting volume of the powdered sample V_s is the difference between the total volume V_{tot} and the volume of the liquid V_{fl}.

$$V_s = V_{tot} - V_{fl}$$

Equation 24

$$V_s = \frac{m_{1fl}}{\rho_{fl}} = \frac{m_{(fl+s)} - m_s}{\rho_{fl}}$$

If we now insert the volume V_s into the initial equation $\rho_s = m_s/V_s$, the following follows after transformation*

Equation 25

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{1fl} - m_{(fl+s)} + m_s} \quad \text{or} \quad \rho_s = \rho_{fl} \times \frac{m_2}{m_1 + m_2 - m_3}$$

$$^*\rho_s = \frac{m_s}{V_s} = \frac{m_s}{\frac{m_{1fl}}{\rho_{fl}} - \frac{m_{(fl+s)} - m_s}{\rho_{fl}}} = \frac{m_s}{\frac{m_{1fl} - (m_{(fl+s)} - m_s)}{\rho_{fl}}} = \frac{\rho \times m_s}{m_{1fl} - (m_{(fl+s)} - m_s)}$$

with masses m_1, m_2 and m_3 in the order of the steps to be performed:

Masses	
m_1	Mass of the liquid in the pycnometer completely filled with liquid
m_2	Mass of the sample material
m_3	Mass of sample and liquid in the filled pycnometer

This means that it is also possible with the aid of this method to replace the volume measurement for density determination "in a roundabout way" by several mass determinations.

Other Methods for Density Determination

There are other methods for density determination, which are also based on Archimedes' principle. Using two solids of different densities (e.g. two weights of different metals), the density of air can be determined.

Finally, the density can be determined by the attenuation of radioactive radiation by the material under investigation.

The absorption of radiation depends on the mass absorption coefficient of the material, its layer thickness and its density. If the corresponding quantities and the physical relation are known, the density of a substance can be calculated.

In the case of magnetic samples, magnetic forces are also exploited to determine the densities of solids or liquids.

Oscillating U-tube Method | Vibration Method

The Oscillating U-tube Method is a widely used measuring method for determining the density of homogeneous liquids. This measuring method is not suitable for suspensions or emulsions which, since they consist of several phases, may segregate.

The sample to be examined is set into mechanical oscillations in a measuring cell (usually a glass tube bent into a U-shape). The physical relation between the natural frequency of the vibration and the mass of the vibrator (U-tube filled with sample) is used to calculate the sample density. The instruments must be calibrated with liquids of known density and viscosity similar to the sample liquid.

Suspension Method

The levitation methods use the principle of Archimedes, namely the special case of levitation in which the densities of the liquid and the solid suspended in it are equal.

Solid densities can be determined by adjusting the density of a test liquid so that the sample reaches a state of suspension. The density of the test liquid can be adjusted by changing the temperature – with precise knowledge of the density of the test liquid as a function of temperature. The density setting of the liquid can also be determined by mixing two liquids of different densities, the density is then determined by the mixing ratio of the liquids or must be determined, for example, by the oscillating U-tube method (see page 17) or the displacement method.

Density gradient column
In density gradient columns, two liquids of different densities are layered on top of each other in a glass tube so that a vertical density gradient (a continuous change in density over the height of the column) is established over time by diffusion. Small solids of different densities then float at different heights. Colored glass spheres of known density are available for calibration.

In addition to small solids (e.g., fibers, powder particles, foil pieces), liquid droplets can also be analyzed for density in density gradient columns – however, the liquid should be insoluble in the liquids of the gradient column.

Streak method
If a capillary tube filled with liquid and running horizontally at the bottom is held in another liquid, the liquid will only flow out of the capillary tube horizontally if the densities of the two liquids match. When the density of the outflowing liquid is lower, streaks form and rise. If the density of the outflowing liquid is higher than that of the surrounding liquid, downward sinking streaks are formed.

Hydrometer

Hydrometers, also known as spindles or hydrometers, are simple measuring instruments for determining the density of liquids or dispersions. These are immersed bodies that float on the surface and immerse to different depths depending on the density of the liquid. The density value of the liquid can be read directly from the immersion depth (the volume of the displaced liquid quantity) on a balance on the hydrometer. For certain applications, there are also hydrometers that do not indicate numerical values for density on their balances, but directly indicate the concentration of sugar (saccharimeter), alcohol (alcoholometer), battery acid or antifreeze in aqueous solution.

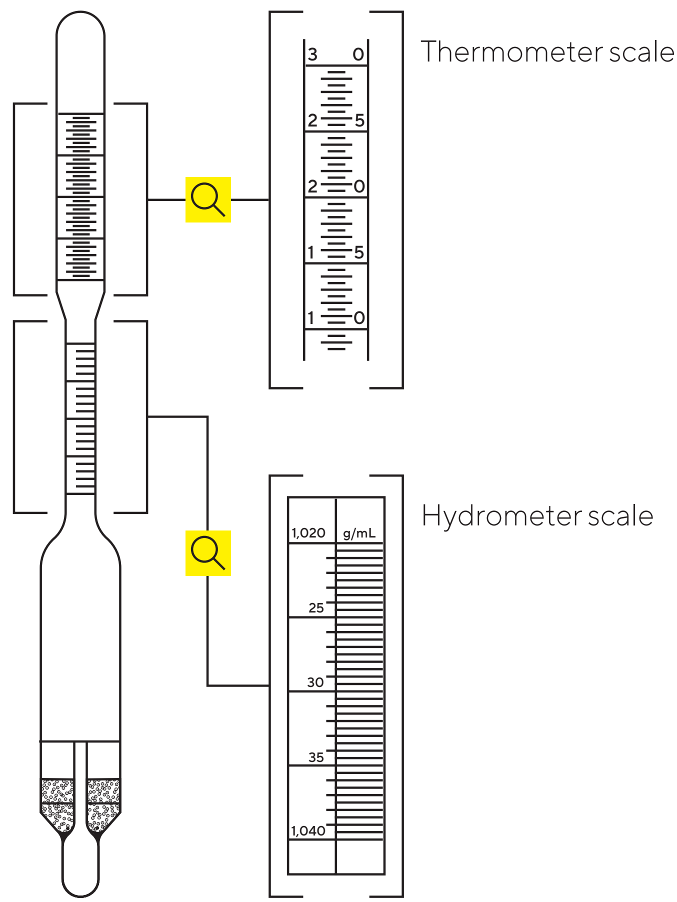


Figure 13: *Special Hydrometer with Integrated Thermometer According to DIN 10290 for Determining the Density of Milk and Skim Milk - the Density Depends on the Fat Content of the Milk*

Application Examples

Determination of the Density of Solids

Special Features of the Samples

Solids are stable in volume and shape at atmospheric pressure. Examples of solids whose density is of interest are metals, glass, or plastics. These solids can consist of only one or several solid phases, one of which can be embedded in another (glass-fiber-reinforced plastic) or the solid phases can be firmly “interlocked” with each other – like the many small crystals in a homogeneous metallic material.

When taking samples for density determination, it is important to ask whether density is to be determined as a material property or whether density determination is to be used as a simple method for searching for material defects – e.g. cracks or cavities in the material. The selection of the measurement method also depends on this.

Suitable Density Determination Methods

Density determination methods suitable for solids are the buoyancy and displacement methods based on Archimedes' principle, and levitation methods can also be used. A prerequisite for the buoyancy or displacement method is working with a buoyancy liquid that does not react with the immersed sample, but wets it as well as possible.

The levitation process is widely used in the glass industry, for example: The glass samples are placed in an organic liquid on which they float at room temperature. Since the temperature dependence of the density of the liquid used is approximately 100 times greater than the temperature dependence of the density of the glass, the glass sample can be made to float by slowly increasing the temperature of the system and thus the density of the glass can be determined.

Performing a Density Determination According to the Displacement Method

Required tools and equipment

- Balance
- Thermometer
- Stand with holding device for the sample
- Beaker with buoyancy liquid of known density – distilled water for all materials that do not react with water

Sample preparation, test performance and evaluation

A beaker with liquid is placed on the balance and the holding device for the sample is immersed in the liquid to the same depth as when weighing with the sample. The balance is tared.

- The test sample is placed next to the beaker on the weighing pan. The mass of the sample body in air $m_{(a)}$ is determined.
- The sample is placed in the holding device and immersed in the liquid. The balance directly displays the mass of displaced liquid m_{fl}.

The density of the sample ρ is calculated according to

$$\rho = \rho_{fl} \frac{m_{(a)}}{m_{(fl)}}$$

Determination of the Density of Porous Solids

Special Features of the Samples

In the case of porous materials, density is closely related to terms such as solid density and pure density, bulk density, total porosity, open porosity or closed porosity.

Porous solids consist of one or more solid phases and the pores. Pores are cavities filled with air (or another gas). They are either located between individual crystals of the solid or enclosed as gas bubbles in glass phases, i.e. non-crystalline rigid melt phases. Thus, there are basically two types of pores: open and closed. Of the open pores, in turn, only a part belongs to the so-called flow-through pores, or another part to the impregnatable pores – with these designations, additional information on the type of impregnation medium and the ambient conditions is always required (e.g. water at a temperature of 22 °C and a pressure of 2,500 Pa).

One speaks of pores when the "diameter" is in the order of 1 nm to 1 mm – dimensions > 1 mm are referred to as cracks or voids, < 1 nm are voids in the crystal lattice. Pores are important components of the structure of many different materials; their quantity, type, shape, orientation, size and size distribution have a major influence on important material and end product properties, e.g. the frost resistance of roof tiles or the insulating properties of sand-lime bricks or aerated concrete. Mechanical strength or corrosion resistance are also influenced by pores in the material.

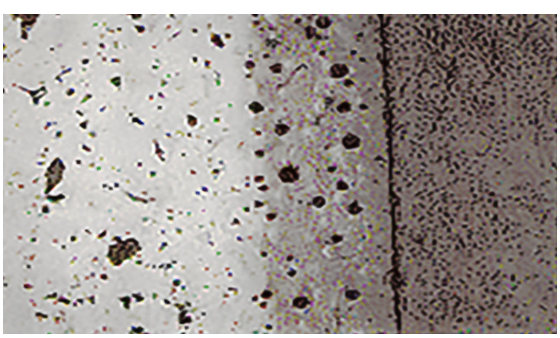


Figure 14: *Micrograph of a Porcelain Plate, Approx. 80x Magnification.*

Note: On the left the porcelain with irregularly shaped pores between different phases, on the right the glaze layer melted during firing with closed spherical pores (bubbles), on the far right synthetic resin as embedding medium for the micrograph preparation

When determining the density, a distinction must be made between whether the density of the pure solid is to be determined or the density of the solid including solid and pores; finally, the porosity may also be of interest.

- The pure solid density (not the solid density) is simply called "density", formerly often referred to as "pure density": $\rho_t = m / V_{solid}$. The pores are therefore not included in this density value.
- The bulk density is the quotient of the mass and the total volume of the sample: $\rho_b = m / V_b$. The bulk density is an average density of the density of the solid and the Gas contained in the pores.
- Open porosity is the volume ratio of open pores to the total volume of the porous body in percent: $\pi_a = V_a / V_b$.
- Closed porosity is the volume ratio of closed pores to the total volume of the porous body in percent: $\pi_f = V_f / V_b$.
- The total porosity is the ratio of the volume of all pores to the total volume of the material in percent: $\pi_t = V_t / V_b$.
- The total porosity is the sum of open and closed porosity: $\pi_t = \pi_a + \pi_f$.

Suitable Density Determination Methods

Suitable density determination methods for porous solids include both the buoyancy and displacement methods. The determination of the solid density can – after grinding the samples up to an average grain size in the order of the pore size - also be carried out by the pycnometer method.

If the bulk density of porous solids is to be determined, the test samples are sometimes covered with a wax or latex sheath (see e.g. DIN EN ISO 2738) so that open pores cannot absorb any liquid during the test. Subsequently, a density determination can be carried out according to the buoyancy method.

Performing a Density Determination According to the Buoyancy Method (Following EN 993-1)

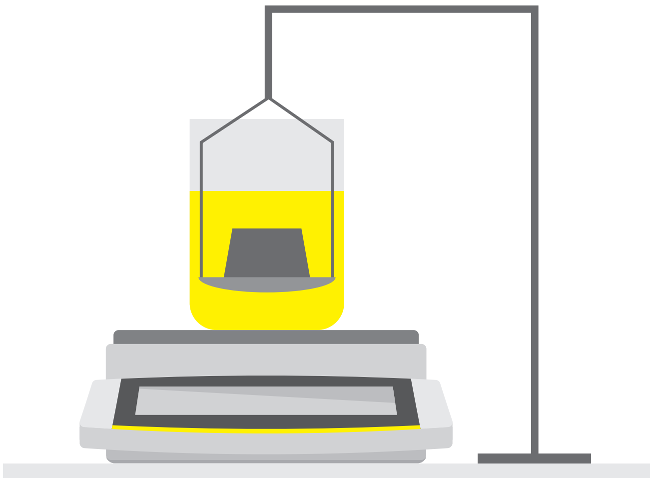


Figure 15: *Experimental Setup for Density Determination Using the Buoyancy Method With the Sartorius Density Set*

Required tools and equipment
- Drying oven at a temperature of (110 ±5) °C
- Balance – error limit of 0.01 g
- Bar to be placed over the weighing pan – density determination set
- Vacuum device with reducible pressure and pressure gauge
- Thermometer with error limit of 1 °C
- Impregnating liquid – distilled water for all materials that do not react with water
- Desiccator

Sample preparation
The shape and size (total volume between 50 cm^3 and 200 cm^3) of the samples and the number of samples to be tested are specified in the standard.

Experimental performance and evaluation
First, the test samples are dried to mass constancy in the drying oven, then cooled in the desiccator to room temperature and then, the mass is determined by weighing in air. $\rightarrow m_1$

The test sample is evacuated under precisely defined conditions and then impregnated with the impregnation liquid (in a vacuum) until the open pores are filled with the liquid in accordance with the test conditions. The apparent mass of the impregnated sample is then determined with a hydrostatic balance. (or using the density set). The sample must be completely immersed in a beaker containing the impregnation liquid as the buoyancy medium. $\rightarrow m_2$

The temperature of the impregnation liquid must be determined.

Finally, the mass of the impregnated sample must be determined by weighing in air. Any liquid adhering to the surface must be removed with a damp sponge before weighing, and the weighing must be carried out quickly so that any loss of mass due to evaporation is excluded. $\rightarrow m_3$

The density of the impregnating liquid ρ_{fl} must be measured or taken dependent on the temperature from a table.

The bulk density ρ_b in g/cm^3 is calculated according to the following equation:

Equation 26

$$\rho_b = \frac{m_1}{m_3 - m_2} \times \rho_{fl}$$

The open porosity π_a in volume percent is calculated as follows:

Equation 27

$$\pi_a = \frac{m_3 - m_1}{m_3 - m_2} \times 100$$

The total porosity π_t is calculated from the values of:

Equation 28

$$\pi_t = \frac{\rho_t - \rho_b}{\rho_t} \times 100$$

The total porosity is the sum of open and closed porosity ($\pi_t = \pi_a + \pi_f$); from this follows for the closed porosity π_f:

Equation 29

$$\pi_f = \pi_t - \pi_a$$

Variables to calculate the density of porous samples	
m_1	Mass of the dry sample
m_2	apparent mass of the soaked sample weighed in liquid
m_3	Mass of the soaked sample weighed in air
ρ_t	Density of the pure solid, determined according to EN 993-2 (or calculated from the composition)
ρ_{fl}	Density of the buoyancy liquid
ρ_b	Bulk density of the sample

In ceramics, the open porosity is often measured not only by the above-mentioned quantities, but also by the so-called Water absorption. The water absorption in percent is calculated from the mass difference of the liquid-soaked sample and the dry sample in relation to the mass of the dry sample. The numerical value is used, among other things, as a comparative value to classify ceramic materials in "dense" and "porous".

Additional information on the type and size distribution of the pores can be obtained with the aid of the mercury pressure porosimeter: The porous samples are filled with mercury under pressure, the pressure is increased in certain steps so that in each case – depending on the pore diameter – a certain amount of pores is filled with mercury. From this, statements can be made about the proportion and diameter of open pores accessible from the outside.

Another method for determining pore fraction, pore shape and size is the image analysis, the quantitative statistical evaluation of polished sections, similar to that shown in Figure 14, under the microscope.

Determination of the Density of Powders and Granules

Special Features of the Samples

A powder is understood to be a "cluster of particles, usually with dimensions smaller than 1 mm".

A granulate consists of coarser particles than a powder the term granules is used in different ways:
granules are – depending on the specialty –

- materials consisting of "secondary" particles formed from agglomerated fine powder particles ("primary particles"),
- Material – for example, intermediate products in the field of plastics or enamel – that has cooled rapidly from the melt phase and may have a drop-like shape.

Suitable Density Determination Methods

Only the pycnometer method can be used to determine the density of powders and granules.

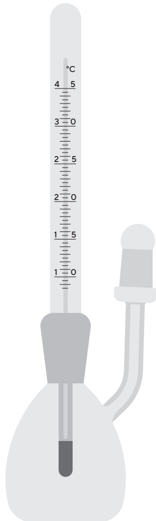


Figure 16: *Pycnometer With Integrated Thermometer*

Carrying out a Density Determination According to the Pycnometer Method (Based on DIN EN ISO 18753)

Required tools and equipment

- Distilled water and another auxiliary liquid e.g. ethanol
- Pycnometer with thermometer and side arm with ground glass stopper
- Water bath
- Vacuum pump
- Balance with error limit of 0.0001 g

Experimental performance and evaluation

The carefully cleaned and dried pycnometer is filled with distilled water and evacuated under defined conditions and then tempered in the water bath to an accuracy of ±0.1 K. The pycnometer is then used to measure the temperature of the water. Then the pycnometer is finally filled. From the mass of the water at the test temperature, the volume of the pycnometer is calculated: $V_{Pycnometer} = m_{Water} / \rho_{Water}$

The pycnometer is dried again and now filled with ethanol according to the procedure described above and finally weighed. The mass of the ethanol and the volume of the pycnometer can then be used to accurately determine the density of the ethanol at the test temperature:
$\rho_{Ethanol} = m_{Ethanol} / V_{Pycnometer}$

$m_{Ethanol} = m_1$

Weigh into the cleaned and dried pycnometer* about 10 g (for powder densities between 2.5 and 4 g/cm³) of the powder dried at 10 K below its decomposition temperature. $\rightarrow m_2$

The pycnometer is now filled with some ethanol so that the powder is moistened, then evacuated again and shaken so that as many air bubbles as possible can escape. The pycnometer is further filled with ethanol, brought to the test temperature and then finally filled up. The total mass of powder and ethanol is determined. $\rightarrow m_3$

The evaluation is performed according to the formula

$$\rho = \rho_{Ethanol} \times \frac{m_2}{m_1 + m_2 - m_3}$$

the density of the sample substance is specified to the nearest 0.001 g/cm³.

* When using modern Sartorius balances with integrated software for density determination, this – time-consuming – process of drying in the drying oven and subsequent cooling in the desiccator can be omitted by using a second, applicative tare reservoir, thus considerably simplifying the work in the laboratory.

Determination of the Density of Homogeneous Liquids

Special Features of the Samples

Homogeneous liquids are relatively simple systems; unlike dispersions, they are always single-phase systems. Mixtures of substances, for example alcohol and water or sugar and water, are solutions of one substance in the other. Authenticity Solutions are always transparent, the particles of the dissolved substance are present as molecules or ions in the solvent.

The density of a solution depends on the concentration of the dissolved substance, i.e. if the relation is known, concentration data can be calculated from density determinations.

The densities of most liquids at 20 °C range from 600 kg/m^3 and 2,000 kg/m^3 or 0.6 g/cm^3 to 2.0 g/cm^3. The densities of liquids are much more dependent on temperature than those of solids. This means that density measurement always includes temperature measurement and | or careful tempering of the sample.

Suitable Density Determination Methods

Various methods are suitable for determining liquid densities: Hydrometers, pycnometers, oscillating meters, the buoyancy and displacement method. The selection of the method depends, among other things, on the required accuracy and the available sample quantity.

Performing a Density Determination According to the Buoyancy Method

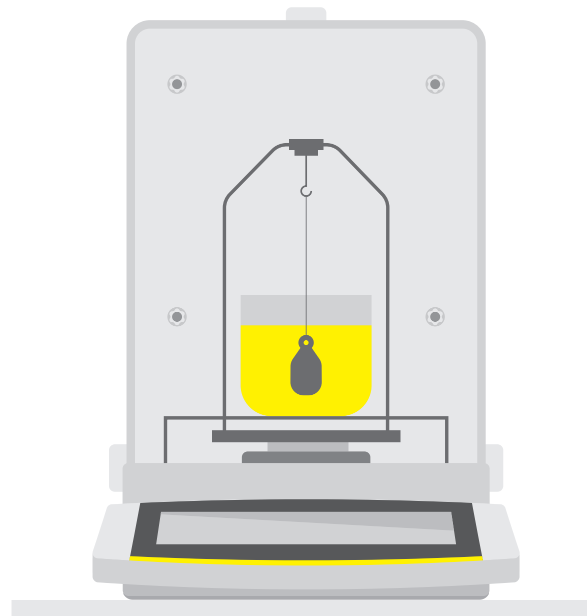


Figure 17: *Determination of Liquid Density by the Buoyancy Method*

Required tools and equipment
- Balance
- density set
- Immersible body with known volume (10 cm^3 for the Sartorius density set, see Fig. 17)
- Thermometer
- possibly a water bath for thermostating the sample

Sample preparation, test performance and evaluation
- Align the empty beaker on the bridge, suspend the immersion body from the frame of the density determination set.
- Taring the balance with the submersed body attached.
- Pour the liquid to be measured into the beaker until the liquid level is 10 mm above the glass body.
- The (negative) measured value displayed by the balance corresponds to the buoyancy of the vitreous body in the sample liquid.
- By dividing the measured value by the volume of the immersed body, one obtains the density to be determined

$$\rho = \frac{m_{fl}}{V_{TK}}$$

Determination of the Density of Dispersions

Special Features of the Samples

Disperse systems or dispersions are mixtures of two or more phases that are insoluble in each other. One phase is continuously coherent – the so-called dispersant – the other phase (or phases) are finely dispersed in the dispersant in the form of tiny isolated particles. The particle sizes of the so-called colloid-disperse systems are generally between 1 µm and 1 nm. When the size of the dispersed particles is >1 µm, we speak of coarse-dispersed systems, and when the size of the particles is <1 nm, we speak of molecular-dispersed systems. There are any number of examples of dispersions, but the term dispersion is the umbrella term for all systems – independent of the state of aggregation of the phases involved.

Dispersed sample types	
Suspensions	Mixtures of solid particles in a liquid Examples: "Dispersion" paints, ceramic casting compounds, scouring powders, toothpaste, ink...
Emulsions	Mixtures of two liquids that are insoluble in each other, one of which is finely distributed in the other in form of small drops Examples: Creams, lotions, mayonnaise, milk, the classic salad dressing from oil and vinegar...
Foams	Mixtures of small gas bubbles in a liquid (or solid)
Mist	Mixtures of small liquid droplets in a gas phase
Smoke	Mixtures of solid particles in a gas phase

Somewhat problematic is the use of the term “stability” in connection with dispersive systems, since these are, strictly speaking, unstable systems. This is reflected in their need for segregation. However, “stable” suspension or emulsion is often used to describe systems that exhibit constant properties over certain periods of time.

Suitable Density Determination Methods

Depending on the consistency of the sample material, density determination methods can also be considered, as they are also used for liquids or solids. Density determination by the oscillating U-tube method is not so well suited, since the many phase boundaries interfere, the density determination is slightly influenced by viscosity in this measurement method, and, in addition, vibration during the measurement can favor phase separation. This method does not determine an average value for the entire sample.

When working with hydrometers, which is possible in principle, care must be taken to ensure that the suspension or emulsion does not show any signs of segregation.

Corresponding to the density determination of liquids or powders, pycnometers can also be used for disperse systems. Depending on the industry, various vessels are used, which are filled to the brim (or to a mark) with the sample substance and weighed out. Here, the possible segregation of the sample during the measurement does not lead to a measurement error. For many suspensions and emulsions, the buoyancy or displacement method is suitable – here, too, care must be taken that the dispersion does not segregate and that the flow behavior of the suspension allows the immersion sample to be immersed quickly enough.

Performing a Density Determination According to the Displacement Method

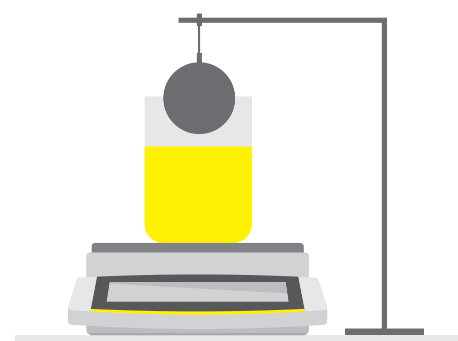


Figure 18: *Density Determination According to the Displacement Method With Gamma Sphere as Immersion Body, Suspended from a Tripod Standing Next to the Balance*

Required tools and equipment
- Balance
- Immersible body with known volume
- Tripod
- Thermometer
- Possibly a water bath for thermostating the sample

Sample preparation, test performance and evaluation
Pour the (tempered) sample into a beaker. Place the beaker on the balance and tare to zero. Lower the immersion unit suspended from the tripod into the sample substance and immerse it up to the mark. The balance shows the mass of the displaced liquid directly (see page 13). Dividing the measured value by the volume of the immersion body yields the density of the sample substance

$$\rho = \frac{m_{fl}}{V_{TK}}$$

Error and Accuracy of Density Determination

In the two preceding chapters (buoyancy method and displacement method), the fundamentals of the two hydrostatic density determination methods were explained in general terms and the formulas for calculating the density were derived.

If high measurement accuracies are required, the real measurement conditions must be taken into account. Generally, the balance does not directly indicate the mass of the samples - as indicated in the general formulas – but their water value in air. Therefore, the weight values corrected for air buoyancy must be inserted in the basic equations.

In addition, for the buoyancy method, the influence of the buoyancy due to the deeper immersion of the sample holder after the liquid has risen by immersion of the sample must be taken into account.

In general, when carrying out density determinations, care must be taken, especially to ensure that the temperature remains constant during the test.

When the solid is immersed in the liquid, no air bubbles should get into the liquid – if they stick to the solid, they falsify its measured value.

Air Buoyancy Correction

For more accurate density determinations, it must be taken into account that the balances do not directly determine the mass of the sample, but its weight value. However, this is dependent on the pressure and temperature dependent air density and must be corrected for the influence of buoyancy in air.

The relation between the mass of the solid m and its weight in air W, i.e. taking into account the buoyancy of the air on the material to be weighed, is generally as follows (ρ_G = density of the normals)

Equation 30

$$m = W \times \frac{1 - \frac{\rho_a}{\rho_G}}{1 - \frac{\rho_a}{\rho}}$$

Displacement Procedure

If this equation is used in the relation to the calculation of the density according to the displacement method, the equation for the determination of the solid density follows, taking into account the air buoyancy (derivation see appendix, page 39)

Equation 31

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{fl}} = (\rho_{fl} - \rho_a) \times \frac{W_s}{W_{fl}} + \rho_a$$

In this form, the density is calculated in the integrated software of the Sartorius balance.

Buoyancy Method

If we insert the relation

$$m = W \times \frac{1 - \frac{\rho_a}{\rho_G}}{1 - \frac{\rho_a}{\rho}}$$

for $m_{(a)}$ and $m_{(fl)}$ into the equation for density determination according to the buoyancy method

$$\rho_s = \rho_{fl} \times \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$$

after mathematical transformation, we get the following formula

Equation 32

$$\rho_s = (\rho_{fl} - \rho_a) \times \frac{W_{(a)}}{W_{(a)} - W_{(fl)}} + \rho_a$$

for calculating the density of solids considering air buoyancy.

Pycnometer Method

Taking into account the air buoyancy correction, the formula for the density determination is as follows

Equation 33

$$\rho_s = (\rho_{fl} - \rho_a) \times \frac{W_2}{W_1 + W_2 - W_3} + \rho_a$$

This formula is used to evaluate the measured values in the integrated software of the Sartorius balances.

Correction of the Buoyancy of the Sample Holder

Buoyancy Method

In addition to the air buoyancy, exact measurements must also consider the additional lift of the suspension, which results from the fact, that the rise of the liquid after the Immersion of the sample immerses the wires deeper into the liquid than before immersion of the sample. (The buoyancy of the sample holder – if the balance is tarred with the holder immersed in the liquid – is not included in the calculation of the density)

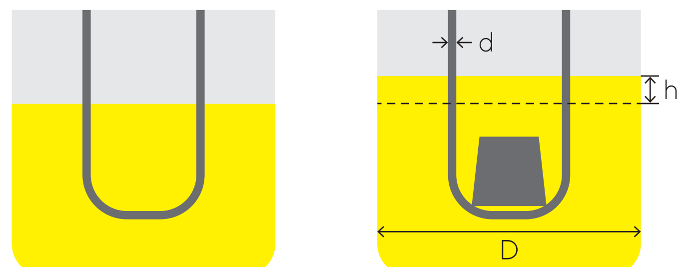


Figure 19: *Sketch for Calculating the Buoyancy Due to the Liquid Rise After Immersion of the Sample in the Buoyant Liquid*

The experimental arrangement with the density determination sets in the buoyancy method results in the wires of the suspension of the sample holder dipping deeper than originally due to the rise of liquid in the vessel. That is, as "more" wire dips, there is additional buoyancy that can be calculated and must be taken into account when correcting the result.

The volume by which the liquid in the container increases with diameter D corresponds to the volume V_{fl} in Figure 19

Equation 34

$$V_{fl} = \frac{\pi \times D^2}{4} \times h$$

The volume V_s of the sample is

Equation 35

$$V_s = \frac{m_{(a)} - m_{(fl)}}{\rho_{fl}}$$

These two volumes are identical, $V_{fl} = V_s$. By inserting the above-mentioned relations resolving them by h, the height of the liquid rise, we obtain

Equation 35

$$h = \frac{(m_{(a)} - m_{(fl)}) \times 4}{\rho_{fl} \times \pi \times D^2}$$

The buoyancy force F_{BD} acting in the liquid layer h on two wires of diameter d is

Equation 37

$$F_{BD} = \rho_{fl} \times V_D \times g = \rho_{fl} \times (2 \times \frac{\pi \times d^2}{4} \times h) \times g$$

and after inserting h, we obtain

$$F_{BD} = \rho_{fl} \times 2 \times \frac{\pi \times d^2}{4} \times \frac{(m_{(a)} - m_{(fl)}) \times 4}{\rho_{fl} \times \pi \times D^2} \times g$$

Equation 38

$$F_{BD} = \frac{\rho_{fl} \times 2 \times \pi \times d^2 \times (m_{(a)} - m_{(fl)}) \times 4 \times g}{4 \times \rho_{fl} \times \pi \times D^2}$$

$$= 2 \times \frac{d^2}{D^2} \times (m_{(a)} - m_{(fl)}) \times g$$

That is, the buoyancy force caused by the wires is proportional to the diameter ratio of wire and beaker.

In addition to the buoyancy of the sample – which is to be determined – the measured value contains a portion of the "wire buoyancy"; the wire buoyancy must therefore be subtracted from the measured value in order to calculate the buoyancy of the sample F_{BS}(corr) alone:

Equation 39

$$F_{BS}\,(corr) = \left[(m_{(a)} - m_{(fl)}) - (2\frac{d^2}{D^2} \times (m_{(a)} - m_{(fl)})\right] \times g$$

$$F_{BS}\,(corr) = \underbrace{(1 - 2\frac{d^2}{D^2})}_{\text{Correction factor}} \times (m_{(a)} - m_{(fl)}) \times g$$

If the diameter of the wire and beaker is known, the factor by which the measured w ell of buoyancy must be multiplied can be calculated. In the Sartorius sealing sets, the wire diameter is d = 0.7 mm, the beaker diameter D = 7.6 mm, and the sample holder has 2 wires.

The correction factor is then

$$Corr = 1 - 2\,\frac{d^2}{D^2} = 1 - 2\,\frac{0.7^2}{76^2} = 0.99983$$

The smaller the wire diameter d, the larger the beaker diameter D and the lower the number of wires of the suspension, the closer the correction factor is to the numerical value 1, i.e. the correction can then be neglected. These conditions can easily be taken into account in the density Determination in underfloor weighing operation.

Back to the formula for density determination using the buoyancy method: Taking into account the buoyancy correction for the wires from

$$\rho_s = \rho_{fl} \times \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$$

$$\rho_s = \rho_{fl} \times \frac{m_{(a)}}{[m_{(a)} - m_{(fl)}] \times Corr}$$

or under simultaneous consideration of the air buoyancy:

Equation 40

$$\rho_s = (\rho_{fl} - \rho_a) \times \frac{W_{(a)}}{[W_{(a)} - W_{(fl)}] \times Corr} + \rho_a$$

This formula for density determination is used by the software integrated in Sartorius balance. The preset values of the standard air density of $\rho_a = 0.0012\ g/cm^3$ and the correction factor of 0.99983 can be used for the Sartorius density set. Alternatively, corrections calculated by the user can be applied.

Displacement Method

Errors due to the additional buoyancy caused by immersion of the holding device for the sample body or threads or wires of the suspension can be eliminated from the outset by immersing them exactly as far into the liquid during empty weighing or taring of the liquid container as later during weighing with the sample body.

In addition, the appropriate experimental conditions also make the correction factor ≈1 (large vessel diameter, small diameter of the holder, if possible only one holder). The equation for calculating the solid density by the displacement method – taking into account air buoyancy and correction factor for sample suspension – is:

Equation 41

$$\rho_s = (\rho_{fl} - \rho_a) \times \frac{W_{(a)}}{W_{(fl)} \times Corr} + \rho_a$$

This is the formula used in Sartorius balance integrated software. The basic setting has the numerical value of 1.0 for the correction factor and of ρ_a = 0.0012 g/cm³ for the air density. User-calculated corrections can also be applied.

When the displacement method is used to determine the density of liquids, including paints and varnishes, metal immersion spheres (gamma sphere) are commonly used which have a taper on the rod (see fig. 20), up to the center of which the nominal volume of the immersion body is calculated. There are designs for different orders of magnitude of the surface tension of the liquids to be tested. The influence of the liquid bulge forming around the rod is taken into account.

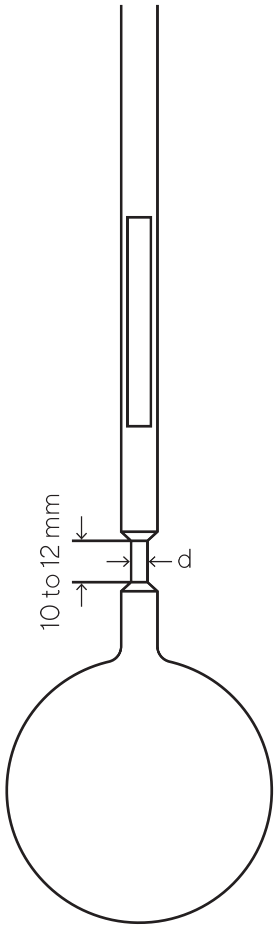


Figure 20: *Immersion Sphere According to DIN EN ISO 2811 Part 3 for the Determination of the Density of Paints and Varnishes According to the Displacement Method*

Avoidance of Systematic Errors

Hydrostatic Procedure

In order to keep the errors in density determination with the various hydrostatic methods as low as possible, particular attention should be paid to the following:

- The temperature must be kept constant throughout the experiment. For water as a buoyancy medium, for example, a temperature change of 0.1 °C causes a density change of 0.0002 to 0.0003 g/cm³, for alcohols of ≈0.0001 g/cm³.
- The balance must be loaded exactly in the center to keep the corner load error as low as possible. When weighing in air using the Sartorius sealing set with the sample receptacle at the top of the frame, the shape of the frame and the application of force at two points on the outside of the base plate of the frame result in a greater torque when the sample is positioned off-center than when using the "normal" weighing pans and the same position deviating from the center.
- After immersion in the liquid, no air bubbles on the sample itself or on the sample holder may be stuck. These create additional buoyancy and thus falsify the weighing result. To avoid this, the sample can be moistened in a separate vessel before the actual measurement – or in an ultrasonic bath.
- An error due to adhesion of liquid to the wire of the sample holder can be neglected by taring the balance before measurement with the sample holder immersed.
- Air buoyancy causes an error in density of ≈0.0012 g/cm³ (corresponding to air density at normal conditions) – therefore, the formulas that take air buoyancy into account should be used to calculate density (see page 26).
- Now that the sample is immersed in the vessel, the liquid level rises so that the wires of the sample holder create additional buoyancy. Depending on the diameter of the vessel used and the diameter and number of wires of the holder, this additional buoyancy can be corrected. (see page 26).

Pycnometer Method

In order to keep the errors in density determination with the pycnometer method as low as possible, particular attention should be paid to the following:

- The temperature must be kept constant during the entire test procedure, or the samples must be carefully thermostatted. For water as an auxiliary liquid, for example, a temperature change of 0.1 °C causes a density change of 0.00002 to 0.0003 g/cm^3, for alcohols of ≈0.0001 g/cm^3.
- There must be no air bubbles in the auxiliary liquid or on the sample itself.
- Air buoyancy causes an error in density of ≈0.0012 g/cm^3 (corresponding to air density at normal conditions) – therefore, the formulas that take air buoyancy into account should be used to calculate density. („Air Buoyancy Correction" see page 26.)

With careful work, the pycnometer method is a measuring method that can be used to determine material densities very accurately.

Error Calculation

Working carefully and avoiding the systematic errors mentioned above, it is possible to calculate the error of density determination according to the rules of error propagation. The error of the density $\Delta\rho$ is mainly based on the measurement errors of the mass determination. The general rule for determining the total error ΔF of quantities calculated from several measured values applies:

In the case of sums (and differences), the absolute single errors add up quadratically:

Equation 42

$$\Delta F = \sqrt{\Delta F_1^2 + F_2^2 + \ldots}$$

In the case of products (and quotients), the relative single errors add up quadratically. (The relative error is the absolute error related to the measured value):

Equation 43

$$\Delta F = \sqrt{\left[\frac{\Delta F_1}{F_1}\right]^2 + \left[\frac{\Delta F_2}{F_2}\right]^2 + \ldots}$$

Buoyancy method

For the determination of the density of solids by the buoyancy method, the following relation applies:

$$\rho_s = \rho_{fl} \times \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$$

or

$$\rho_s = (\rho_{fl} - \rho_a) \times \frac{W_{(a)}}{[W_{(a)} - W_{(fl)}] \times Corr} + \rho_a$$

Since the correction factor for the sample holder and the density of the air have no noticeable influence on the error of density, they need not be considered in the error calculation.

When the basic rules of error calculation are applied, first the absolute error of the denominator $\Delta[m_{(a)} - m_{(fl)}]$ is calculated

Equation 44

$$\Delta[m_{(a)} - m_{(fl)}] = \sqrt{\Delta m_{(a)}^{2} + m_{(fl)}^{2}}$$

and then the total relative density error $\Delta\rho/\rho$

Equation 45

$$\frac{\Delta\rho}{\rho} = \sqrt{\left[\frac{\Delta\rho_{fl}}{\rho_{fl}}\right]^2 + \left[\frac{\Delta m_{(a)}}{m_{(a)}}\right]^2 + \left[\frac{\Delta m_{(a)} - m_{(fl)}}{m_{(fl)}}\right]^2}$$

The total error of the weighing in air ($\Delta m_{(a)}$) is the sum of reproducibility and a linearity error of one digit - independent of balance type, since differential weighing is involved.

The maximum error of weighing in liquid ($\Delta m_{(a)}$ - $\Delta m_{(fl)}$) is assumed to be on average 10 times larger than that of weighing in air – this assumption is based on extensive measurements for density determination using the buoyancy method.

For the liquid density error, the value of 0.0003 g/cm³ for water or 0.00009 g/cm³ for ethanol is used, i.e. the value corresponding to a misreading of the thermometer by ±0.1 °C or a temperature change during the measurement by ±0.1 °C .

The following diagrams (see Figure 21 to Figure 26) show the relative error of the solid density for the buoyancy method in water or ethanol as the buoyancy medium as a function of the sample size and the density of the sample using the example of some Sartorius balances with different readabilities and ranges.

It is shown, that the error in the density determination is clearly dependent on the density of the sample, the lower the density of the sample, the greater the error of the final result.

When calculating $\Delta\rho/\rho$

$$m_{(fl)} = m_{(a)} \times \left[1 - \frac{\rho_{fl}}{\rho_s}\right]$$

is used for $m_{(fl)}$ for ρ_{fl} a density for water of 1.0 g/cm³ and a density for ethanol of 0.789 g/cm³ is assumed.

Another diagram (see Figure 27) shows the error of the density determination of liquids by the buoyancy method for liquid densities between 0.5 and 2.2 g/cm³ using the glass body from the Sartorius density set.

The immersion body has a volume of (10 + 0.01) cm³, a density of 2.48 g/cm³ and a tolerance of 0.5 mg related to buoyancy in water. Again, the error of the density depends on the density value of the tested sample.

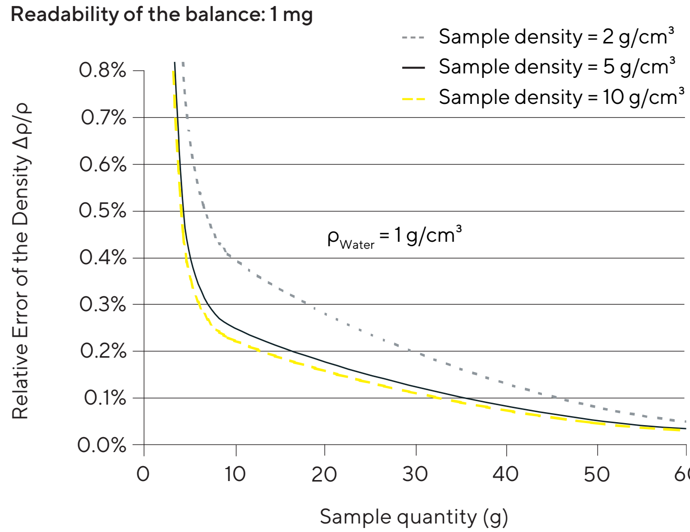


Figure 21: *Solid Density Determination By the Buoyancy Method - Relative Density Error in Dependence of the Sample Size and the Sample Density*

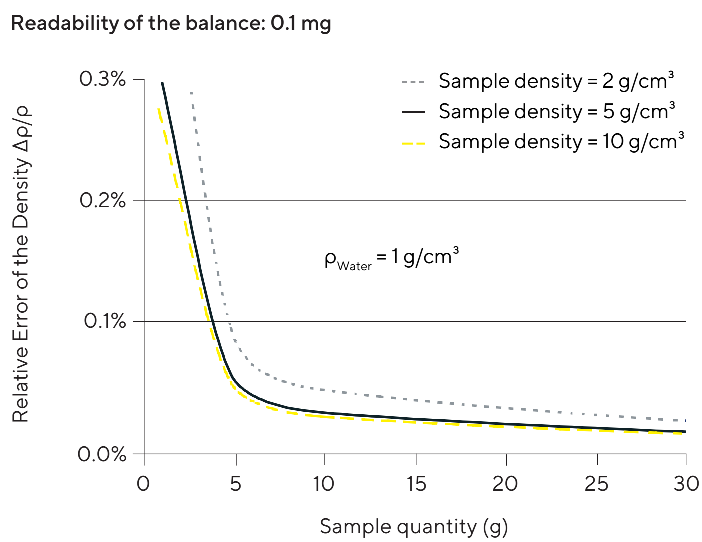


Figure 22: *Solid Density Determination by the Buoyancy Method - Relative Error of Density in Dependence on the Sample Size and the Sample Density*

Readability of the balance: 0.01 mg

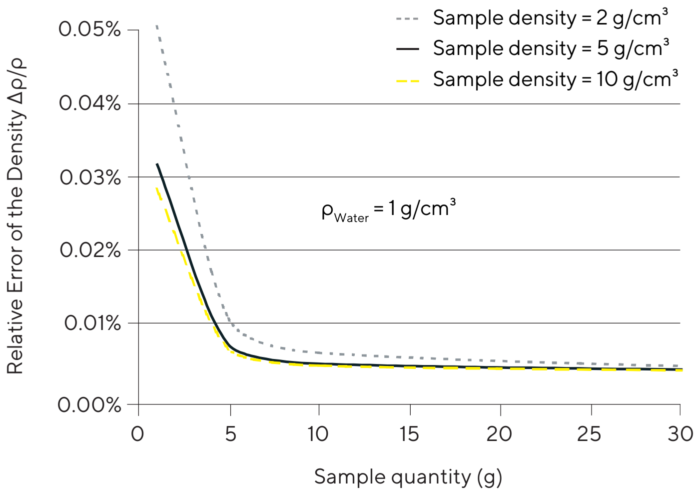


Figure 23: *Solid Density Determination by the Buoyancy Method - Relative Error of Density in Dependence on the Sample Size and the Sample Density*

Readability of the balance: 1 mg

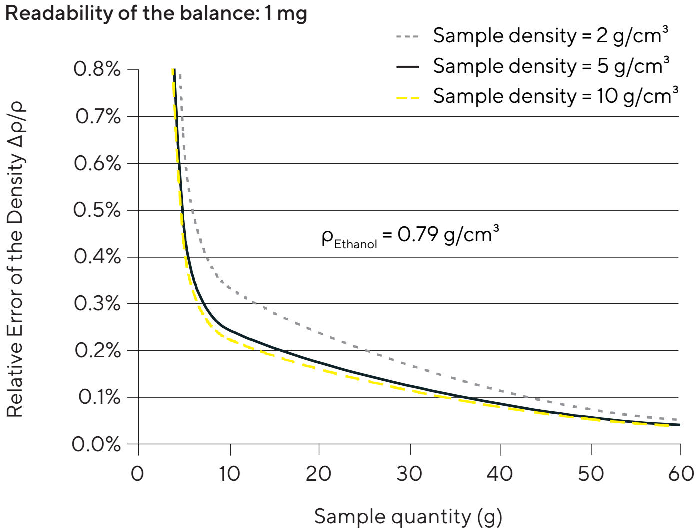


Figure 24: *Solid Density Determination by the Buoyancy Method - Relative Error of Density in Dependence on the Sample Size and the Sample Density*

Readability of the balance: 0.1 mg

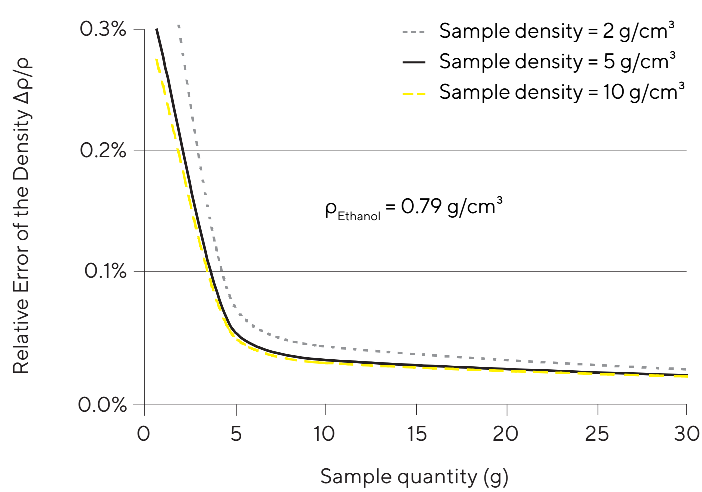


Figure 25: *Solid Density Determination by the Buoyancy Method - Relative Error of Density in Dependence on the Sample Size and the Sample Density*

Readability of the balance: 0.01 mg

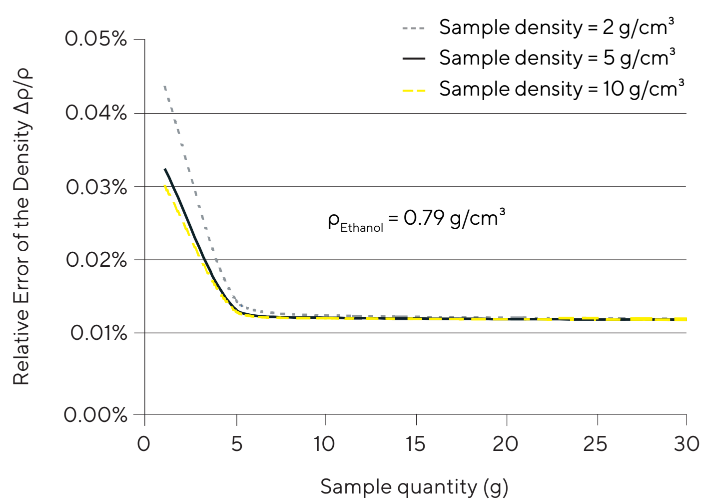


Figure 26: *Solid Density Determination by the Buoyancy Method - Relative Error of Density in Dependence on the Sample Size and the Sample Density*

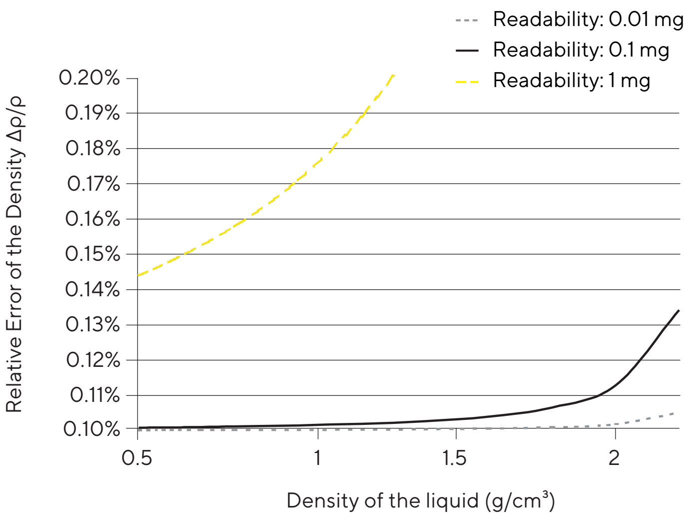


Figure 27: *Liquid Density Determination by the Buoyancy Method - Relative Error of the Density as a Function of the Sample Density and the Readability of the Balance for Density Determination With the Sartorius Density Set*

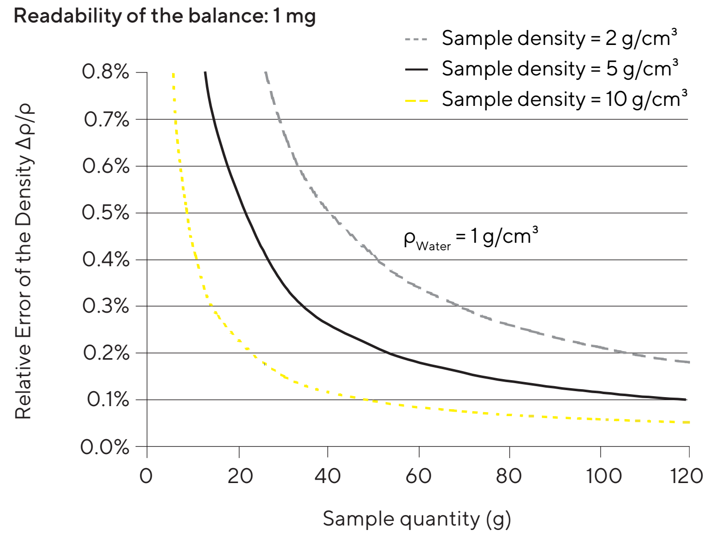


Figure 28: *Solid Density Determination by the Displacement Method - Relative Error of the Density in Dependence of the Sample Size and the Sample Density*

Displacement Procedure

The following relation applies to the determination of the solid density by the displacement method, whereby the air density ρ_a is assumed to be constant and is not included in the error calculation.

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{fl}} = (\rho_{fl} - \rho_a) \times \frac{W_s}{W_{fl}} + \rho_a$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left[\frac{\Delta\rho_{fl}}{\rho_{fl}}\right]^2 + \left[\frac{\Delta m_{(a)}}{m_{(a)}}\right]^2 + \left[\frac{m_{(fl)}}{m_{(fl)}}\right]^2}\ ^*$$

with

$$^*m_{fl} = \frac{\rho_{fl}}{\rho_s} \times m_{(a)}$$

Compared to the buoyancy method, it is noticeable (see Figure 21 to Figure 26) that the results of the solid density determination with the buoyancy method show a smaller error as with the displacement method (see Figure 28). It is also clear that in the buoyancy method the error decreases with increasing sample density, while in the displacement method it increases with increasing sample density.

Pycnometer Method

For density determination by the pycnometer method, the following relation applies

$$\rho_s = \rho_{fl} \times \frac{m_1}{m_1 + m_2 - m_3} \quad \text{or} \quad \rho_s = (\rho_{fl} - \rho_a) \times \frac{W_2}{W_1 + W_2 - W_3} + \rho_a$$

The air density error is again neglected. The total error of the weight value is the sum of the reproducibility of the weighing type and a linearity error of one digit. For the error of the liquid density, the value of 0.00003 g/cm³ is used, i. e. the value corresponding to a misreading of the thermometer by ±0.1 °C – or a change in temperature by ±0.1 °C. The liquid density is used as 1.0 g/cm³ for water as an auxiliary liquid.

First, the error of the denominator $\Delta[m_1 + m_2 - m_3]$ is calculated

$$\Delta[m_1 + m_2 - m_3] = \sqrt{\Delta m_1^2 + \Delta m_2^2 + \Delta m_3^2}$$

and then the total relative density error $\Delta\rho/\rho$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left[\frac{\Delta\rho_{fl}}{\rho_{fl}}\right]^2 + \left[\frac{\Delta m_2}{m_2}\right]^2 + \left[\frac{\Delta(m_1 + m_2 - m_3)}{(m_1 + m_2 - m_3)}\right]^2}$$

The following diagrams (see Figure 29 to Figure 3 1) for balances with different readabilities and weighing ranges show the relative error of the density determination in dependence on the sample quantity and the sample density.

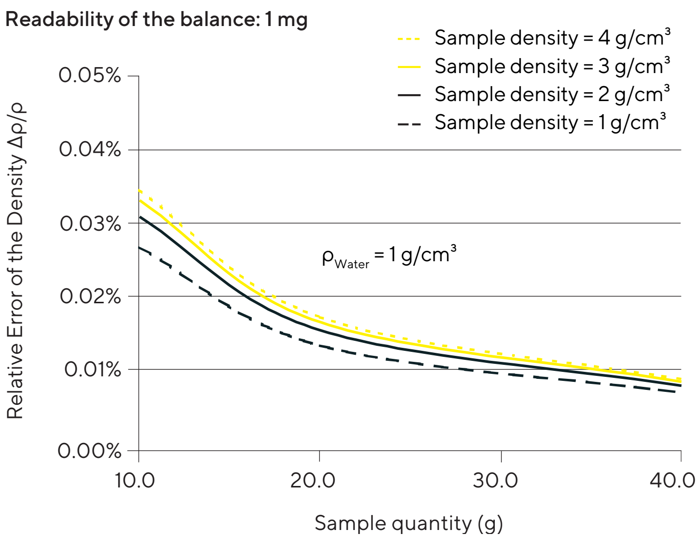


Figure 29: *Density Determination by the Pycnometer Method - Relative Error of Density as a Function of Sample Quantity and Sample Density*

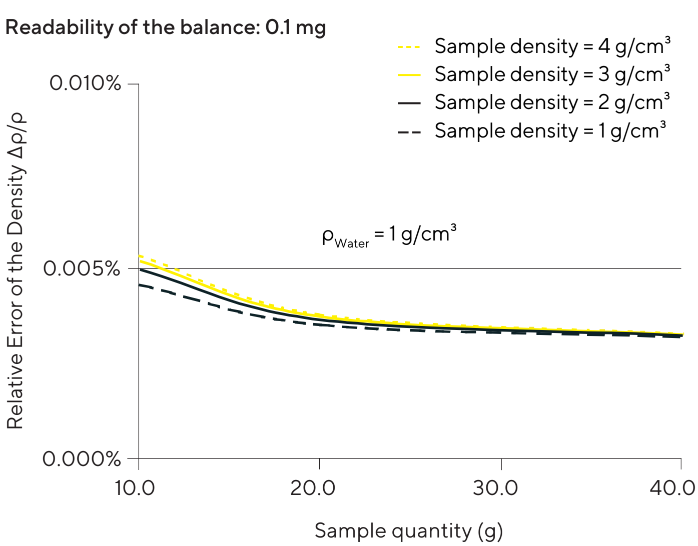


Figure 30: *Density Determination by the Pycnometer Method - Relative Error of Density as a Function of Sample Quantity and Sample Density*

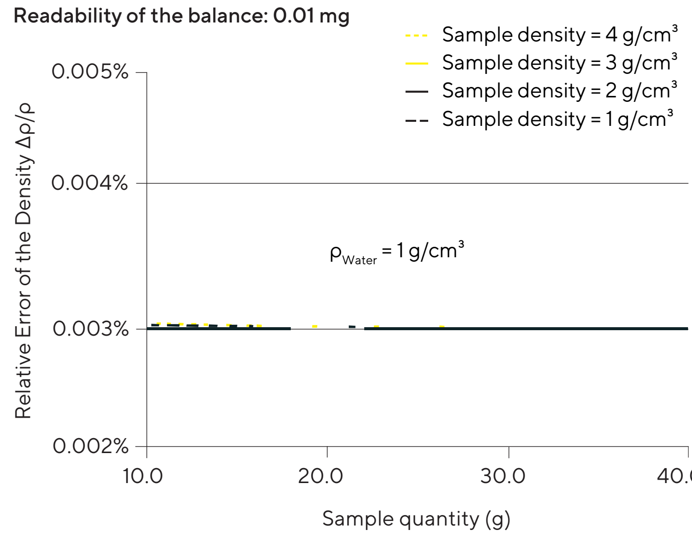


Figure 31: *Density Determination by the Pycnometer Method - Relative Error of Density as a Function of Sample Quantity and Sample Density*

Comparison of Different Methods for Density Determination

On the following pages, different methods of density determination are compared so that the most important advantages and disadvantages of each method can be seen at a glance.

	Buoyancy method	Displacement method	Pycnometer method	Weighing of a defined volume
Suitable for:	▪ Solid state ▪ Liquids ▪ Dispersions ▪ Gases	▪ Solid state ▪ Liquids ▪ Dispersions	▪ Solids ▪ Powders ▪ Granules ▪ Liquids ▪ Dispersions	▪ Powder (bulk density) ▪ Liquids ▪ Dispersions
Advantages	Suitable for almost all sample types	Suitable for almost all sample types	Suitable for all sample types	
	Flexible with regard to the sample size	Flexible with regard to the sample size		
	Balances are already available	Balances are already available	Balances are already available	Balances are already available
	Quick to perform	Quick to perform	Very accurate process	Quick to perform
	Particularly easy test execution due to balances with integrated software with operator guidance and evaluation	Particularly easy test execution due to balances with integrated software with operator guidance and evaluation	Particularly easy test execution due to balances with integrated software with operator guidance and evaluation	
Disadvantages	Tempering of solid state bodies and liquids requires work	Tempering of solid state bodies and liquids requires work	Tempering of solid state bodies and liquids requires work	
	For liquid density determination, large sample volume required!	For liquid density determination, large sample volume required!	Labor-intensive	
		Liquid evaporation to be noted	Time-intensive	
	Wetting of the sample in the liquid to be noted	Wetting of the sample in the liquid to be noted		
	Do not trap air bubbles	Do not trap air bubbles	Do not trap air bubbles	

	Buoyancy method	Displacement method	Pycnometer method	Weighing of a defined volume
Measurement uncertainty*	Depending on the balance used and the sample quantity or the sample density	Depending on the balance used and the sample quantity or the sample density	Dependent on the balance and sample quantity	Dependent on the balance and sample quantity
Readability 1 mg	Solids: $<0.4\%$ for $m > 10$ g Liquids: ** $<0.20\%$ for $\rho = 1.3$ g/cm³	Solids: $<0.2\%$ for $m > 50$g (water), $\rho = 5$ g/cm³	$<0.02\%$ for $m > 20$ g	Liquids, dispersions $<1\%$ for $V = 100$ mL, $<0.1\%$ for $V = 1000$ mL and $\rho < 2$ g/cm³
Readability 0.1 mg	Solids: $<0.1\%$ for $m > 5$ g Liquids: ** $<0.11\%$ for $\rho < 1.8$ g/cm³	Solids: $<0.02\%$ for $m > 50$ g (water), $\rho = 5$ g/cm³	$<0.005\%$ for $m > 10$ g	
Readability 0.01 mg	Solids: $<0.10\%$ for $m > 5$g (water) $<0.15\%$ for $m > 5$g (ethanol) Liquids: ** ~0.1% for $\rho < 1.5$ g/cm³		$<0.003\%$ for $m > 10$ g	

* for more detailed information, see chapter Density determination errors
** with the immersion body of the Sartorius density set

	Hydrometer	Oscillating U-tube Method	Density gradient column
Suitable for:	▪ Liquids (dispersions)	▪ (homogeneous) liquids	▪ Solids (small test samples) ▪ (liquids)
Advantages	Measurement simple	Small sample quantity of approx. 1 mL	Several test samples can be tested simultaneously
	Quick to perform	Quick to perform	
	Low-priced Floating body		
Disadvantages	For dispersions: Measurement error due to segregation of the sample	For dispersions: Measurement error due to segregation of the sample	Laborious test preparation
		Density is minimally influences due to viscosity of the sample	
		Expensive equipment must be purchased	
Measurement uncertainty	0.1 to 10 kg/m³ or 0.0001 to 0.01 g/cm³ for $\rho = 0.6$ to 2.0 g/cm³	There are measuring devices with uncertainties of 0.001 g/cm³, 0.0001 g/cm³ or 0.00001 g/cm³	Calibrated glass comparison bodies with densities from 0.8 to 2.0 g/cm³ ± 0.0002 g/cm³
	Hydrometers are suitable for certain areas of the surface tension of the sample		

Appendix

Temperature Dependence of the Density

Density as a function of temperature can be calculated using the volume expansion coefficient γ. In general, the coefficient of expansion is given only for a certain temperature range (e.g. 20 °C to 100 °C) for which a linear approximation is permissible. Numerical values for γ of different gases and liquids can be found in physical-chemical tables.

For solids, the coefficient of linear expansion is usually given α. For the conversion of the linear into the volume expansion coefficient, the relation $\gamma \approx 3\,\alpha$ applies.

The density of a substance at a temperature T_2 can be calculated with the help of the density at temperature T_1 and the volume expansion coefficient:

$$\rho(T_2) = \frac{\rho(T_2)}{1 + \gamma\,(T_2 - T_1)}$$

Hydrostatic Density Determination – Eliminating the Volume in the Equations for ρ

By completely immersing the solid in the liquid, it is given by the experimental arrangement that the volumes of solid and liquid are equal.

One can then derive a relation between the masses and densities of the two substances in which the volume is no longer explicitly included.

$$\rho_{fl} = \frac{m_{fl}}{V_{fl}} \qquad \rho_s = \frac{m_s}{V_s}$$

transformed:

$$V_{fl} = \frac{m_{fl}}{\rho_{fl}} \qquad V_s = \frac{m_s}{\rho_s}$$

from $V_{fl} = V_s$

$$\frac{m_{fl}}{\rho_{fl}} = \frac{m_s}{\rho_s}$$

To determine the solid density, then follows:

$$\rho_s = \rho_{fl} \times \frac{m_s}{m_{fl}}$$

or for the determination of liquid density:

$$\rho_{fl} = \rho_s \times \frac{m_{fl}}{m_s} = \frac{m_{fl}}{V_s}$$

Air Density Determination

Based on the equations for the relation between mass and weight value for aluminum and for steel, the relation for the determination of the air density is derived:

$$W_{Al} = m_{Al} \times \frac{1 - \frac{\rho_a}{\rho_{Al}}}{1 - \frac{\rho_a}{\rho_N}} \qquad 1 = \frac{m_{Al}}{W_{Al}} \times \frac{1 - \frac{\rho_a}{\rho_{Al}}}{1 - \frac{\rho_a}{\rho_N}}$$

$$W_{St} = m_{St} \times \frac{1 - \frac{\rho_a}{\rho_{St}}}{1 - \frac{\rho_a}{\rho_N}} \qquad 1 = \frac{m_{St}}{W_{St}} \times \frac{1 - \frac{\rho_a}{\rho_{St}}}{1 - \frac{\rho_a}{\rho_N}}$$

Normalizing the equations to 1 and then equating yields:

$$\frac{m_{St}}{W_{St}} \times \frac{1 \frac{\rho_a}{\rho_{St}}}{1 - \frac{\rho_a}{\rho_N}} = \frac{m_{Al}}{W_{Al}} \times \frac{1 - \frac{\rho_a}{\rho_{Al}}}{1 - \frac{\rho_a}{\rho_N}}$$

$$\frac{m_{St}}{W_{St}} \times \left(1 - \frac{\rho_a}{\rho_{St}}\right) = \frac{m_{Al}}{W_{Al}} \times \left(1 - \frac{\rho_a}{\rho_{Al}}\right)$$

$$W_{Al} \times m_{St} \times \left(1 - \frac{\rho_a}{\rho_{St}}\right) = W_{St} \times m_{Al} \times \left(1 - \frac{\rho_a}{\rho_{Al}}\right)$$

$$W_{Al} \times m_{St} - W_{Al} \times m_{St} \times \frac{\rho_a}{\rho_{St}} = W_{St} \times m_{Al} - W_{St} \times m_{Al} \times \frac{\rho_a}{\rho_{Al}}$$

$$W_{St} \times m_{Al} \times \frac{\rho_a}{\rho_{Al}} - W_{Al} \times m_{St} \frac{\rho_a}{\rho_{St}} = W_{St} \times m_{Al} - W_{Al} \times m_{St}$$

$$W_{St} \times m_{Al} - W_{Al} \times m_{St} = \left(\frac{W_{St} \times m_{Al}}{\rho_{Al}} - \frac{W_{Al} \times m_{St}}{\rho_{St}}\right)$$

$$\rho_s = \frac{W_{St} \times m_{Al} - W_{Al} \times m_{St}}{\left(\frac{W_{St} \times m_{Al}}{\rho_{Al}} - \frac{W_{Al} \times m_{St}}{\rho_{St}}\right)}$$

Density Questions

1. How is density defined and what is the unit of density?
2. How does the density change when the temperature increases?
3. Why do bodies appear lighter when weighed in water than in air? – Describe the phenomenon!
4. When do bodies "float" in a liquid? Which statement is then true for the densities of liquid and solid?
5. How does the experimental setup for density determination differ according to the buoyancy method and the displacement method, what does the measured value contain in each case?
6. Which methods can be used to determine the density of liquids? – Why are liquid densities determined and what conclusions can be drawn from the measured values?
7. What is a dispersion and how can its density be determined?
8. What is the difference between density (formerly: pure density) and bulk density - how can one determine the density of porous materials?
9. What is the density of air, when do we need to know the density of air and how do we determine it?

Air Buoyancy Correction

Using the example of density determination by the displacement method, the formula for calculating the density is derived by taking the air buoyancy into account:

The density is calculated according to

$$\rho_s = \frac{\rho_{fl}}{m_{fl}} \times m_s$$

for the mass, the following relation must be inserted, which describes the dependence of the mass on the air density:

$$m = W \times \frac{1 - \frac{\rho_a}{\rho_G}}{1 - \frac{\rho_a}{\rho}}$$

$$\rho_s = \rho_{fl} \times \frac{W_s \times \left(1 - \frac{\rho_a}{\rho_G}\right) \times \left(1 - \frac{\rho_a}{\rho_{fl}}\right)}{\left(1 - \frac{\rho_a}{\rho_s}\right) \times W_{fl} \times \left(1 - \frac{\rho_a}{\rho_G}\right)}$$

$$\rho_s = \rho_{fl} \times \frac{W_s \times \left(1 - \frac{\rho_a}{\rho_{fl}}\right)}{\left(1 - \frac{\rho_a}{\rho_s}\right) \times W_{fl}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \times \frac{\rho_{fl} - \rho_a}{1 - \frac{\rho_a}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a) \times \frac{1}{\frac{\rho_s}{\rho_s} - \frac{\rho_a}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a) \times \frac{1}{\frac{\rho_s - \rho_a}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a) \times \frac{\rho_s}{\rho_s - \rho_a}$$

$$\rho \times \frac{\rho_s - \rho_a}{\rho_s} = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a)$$

$$\rho_s - \rho_a = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a)$$

$$\rho_s = \frac{W_s}{W_{fl}} \times (\rho_{fl} - \rho_a) + \rho_a$$

10. How to determine the density of powders?
11. Which product or material properties can be controlled on the basis of density measurements?
12. What is the significance of density determination in the field of prepackaging regulation?
13. What does the accuracy of a density determination depend on and how do you calculate the error of the density value.
14. Explain the meaning of each quantity in the following formulas, which includes "Corr?"
    $\rho = (W_a \times (\rho_{fl} - L_A)) / ((W_a - W_{fl}) \times Corr) + L_A$
    $\rho = (W_a \times (\rho_{fl} - L_A)) / (W_{fl} \times Corr) + L_A$
    $\rho = (W_a \times (\rho_{fl} - L_A)) / (W_{fl} + W_a - W_r) + L_A$
15. What is the advantage of the "applicative tare memory" of the Sartorius balances in density determination with the pycnometer method?

Notes for Answering the Questions

1. Density = Mass / Volume
   1000 kg/m³ = 1 kg/dm³ = 1 g/ cm³ = 1 g/mL
2. The density decreases.
3. Resultant force = weight force minus buoyancy force. The buoyancy depends on the gravity pressure in the liquid $p = \rho \times g \times h$;
   $F = p \times A = \rho \times g \times h \times A = \rho \times g \times V$ (see page 9)
4. The densities of solid and liquid are the same (see page 11).
5. Vessel with buoyancy liquid stands on the weighing pan in the displacement method, has no contact with the weighing pan in the buoyancy method.
   Measured value corresponds to the mass of the displaced liquid in the displacement method, measured value corresponds to the mass of the body reduced by the buoyancy in the buoyancy method (see page 12 – page 13).
6. Buoyancy method, displacement method, weighing a defined volume, hydrometer, Oscillating U-tube method, levitation method (immiscible liquids). Conclusion on concentration ratios, FPV: gravimetric instead of volumetric dosing, page 24 and table page 35 ff.
7. Multiphase system consisting of a continuous phase (matrix) and one or more finely divided phases (disperse phases). (see page 25) Selection of the method depending on the required accuracy, consistency and flowability of the dispersion: either as for liquids or pycnometer method. Note the influence of segregation in different methods, in the pycnometer method segregation has no influence on the result.
8. For porous materials: Bulk density is related to the total volume including open and closed pores – (Pure) density is related to the pure solid volume. Density determination methods for porous samples: Buoyancy method (see page 21) or pycnometer method for (pure) density determination after grinding the sample to a fine powder; advantage of the pycnometer method: a higher measuring accuracy is possible, existing closed pores are not "added" to the solid.
9. ρ_{Air} = 0.0012 g/cm³
   In the case of analytical and microbalances, the weight value must be corrected in order to specify the mass exactly. With two weights of different density and thus different volumes and approximately the same mass, the air density can be determined with a microbalance. The conventional weighing values of the weights must be known, the material densities must be known, then the air density can be calculated from the measured weight values, see page 13 - included in the software of Sartorius microbalances and ultra-microbalances.
10. According to the pycnometer method, see page 16, see page 23 for the experimental description.
11. Porosity, voids, crystal content (crystalline phases have a higher density than non-crystalline glassy phases), cooling rate in glass, concentration of a constituent in a solution, solid content in suspensions, see page 6.
12. Volumetry is replaced by gravimetry as a more accurate and simpler measuring method. The proportionality factor between mass and volume is density. (Calibrated devices must be used!)
13. Careful adherence to experimental conditions, e.g., tempering see page 29. The error of the measured value depends on the readability, reproducibility of the balance. The total error of the result must be calculated according to the rules of error propagation, see page 30.
14. See operating instructions of the LA and FC balance models.
15. Time saving by omitting the drying process of the pycnometer, see page 23.

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